

Annotation

For more than 30 years K.-R. Biermann had been studying various aspects of the life and work of Gauss. Here, three of his pertinent papers are translated, and the bibliographies appended to them include nine more of such papers or notes. No breakthrough discoveries had occurred, but students of Gauss will find the three chosen papers very helpful since they provide interesting information lacking elsewhere.

Kurt-R. Biermann

Three Papers Concerning Gauss

Translated by Oscar Sheynin

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Introduction

For many years Biermann studied the life and work of Gauss, and it seems therefore proper to publish translations of some of his papers. I add a tiny and barely known piece by Gauss himself which throws some light on the lost German text of the *Theory of Motion* and a few pages of my own which concisely describe the known negative aspect of his life and work.

The works of Gauss are mentioned throughout, and I list them here.

Werke, Bde 1 – 12. Göttingen, 1863 – 1930. Reprint: Hildesheim, 1973 – 1981.

Werke, Ergänzungsreihe, Bde 1 – 5. Hildesheim, 1973 – 1981.

These volumes are reprints of the previously published correspondence of Gauss with Bessel (Bd. 1); Bolyai (Bd. 2); Gerling (Bd. 3); Olbers (Bd. 4, No. 1 – 2); and Schumacher (Bd. 5, No. 1 – 3).

Notation: W-i = *Werke*, Bd. i.

W/Erg-i = *Werke*, Ergänzungsreihe, Bd. i.

Many letters exchanged by Bessel, Gauss, Olbers and Schumacher are quoted.

Notation: B – S = letter from Bessel to Schumacher;

G – O = letter from Gauss to Olbers; etc.

Special notation: **S**, **G**, i means see downloadable Document i on my website www.sheynin.de or on its copy at Google, Oscar Sheynin, Home. Such documents are my translations from Russian or German or in their original English if barely available.

I

On the relations between C. F. Gauss and F. W. Bessel

Über die Beziehungen zwischen C. F. Gauss und F. W. Bessel.
Mitt. Gauss Ges. Göttingen No. 3, 1966, pp. 7 – 20

[1] The most important source needed for judging the relations between Carl Friedrich Gauss, the *princeps mathematicorum*, and Friedrich Wilhelm Bessel, the *great astronomer* of his time, as Alexander von Humboldt had called him, is the correspondence between these spiritual heroes (1880). Being extremely useful for the history of science, it can perhaps be without exaggeration considered the most important of all the published scientific correspondence¹. Its letters contain a wealth of utterly interesting reasoning, results, observations, problems etc. pertaining to the history of science. Not surprisingly, it became sensational once the Berlin Academy of Sciences published it under the editorship of the astronomer Arthur Auwers.

A rarely seen but perceptible astonishment appeared by that time since Gauss had so often remarked about the reports on scientific discoveries, now and then somewhat laconically, that he found them long ago. Nowadays we understand that Gauss was not on any occasion guilty of scientific overstatements and that he had actually been far, far ahead of his time, but, when his correspondence with Bessel had appeared, a large part of the now published materials had still remained unknown.

The friends of Gauss never doubted that his announcements were absolutely truthful. However, already some of those contemporaries, who were more remote from him, sometimes expressed a slight suspicion that the great man from time to time exaggerated [his merits] and did not or could not have accepted the contributions of other scientists².

It is only natural that among the later generation, which had not anymore any personal relations with Gauss, such doubts had been strengthening. Indeed, until then, apart from the exchange of his letters with Sophie Germain published in 1879/1880, only the correspondence of Gauss with Schumacher had appeared, with a man who was hardly a scientific match for Gauss. For this reason no sufficient exoneration of Gauss had been offered, until, as stated above, the publication of the correspondence between Gauss and Bessel paved the way for a change.

Since then the superiority of Gauss over his scientific contemporaries has been demonstrated anew to his astonished readers with each new edition of his notes or letters. Take Weierstraß, a grandmaster of mathematics in his own right, so dissimilar from Gauss. He still had many common features with the latter; for example, he possessed many discoveries which only after some decades became generally known. Once, in a letter to H. A. Schwarz,

he expressed his amazement at the fact *that Gauss, already in the beginning of this century, possessed the main ideas of our present analysis*. Weierstrass referred to a letter from Gauss to Bessel of 18 Dec. 1811 in which the sender clearly formulated the *Cauchy integral theorem* and described its importance³.

However, the correspondence between Gauss and Bessel is so instructive not only in the scientific sense. It is also very important for judging both of them, their human peculiarities and unusual features as well as their interrelations. But exactly in this respect the letters leave many questions open, and, to answer them, we ought to fall back upon the letters exchanged between Gauss' friends. By using this possibility, I am trying here to contribute something to the interpretation of the relations between Gauss and Bessel. I especially draw on the letters between Bessel and Schumacher which until now only Johann Adolf Repsold had consulted when compiling their biographies []. Regrettably, contemporary turmoil deprived these meritorious contributions of due attention.

Gauss and Bessel began to exchange letters in Dec. 1804 while the latter was still a commercial office worker in the Bremen firma Kulenkamp. The correspondence continued during Bessel's work as an assistant in Schröter's observatory in Lilienthal near Bremen (March 1806 – March 1810) and went on while Bessel had been Director of an observatory and ordinary Professor of astronomy in Königsberg (from May 1810). It ended one and a half years before he died on 17 March 1846. Gauss wrote Bessel 75 letters. Only one of them dated 28 Oct. 1843 was not included in their published correspondence (1880) since it was offered at an autograph market. That publication also contains 119 letters from Bessel to Gauss.

[2] On 28 June 1807 Gauss and Bessel became acquainted in Olbers' place in Bremen. Olbers was closely connected with both of them as most tightly with H. C. Schumacher. It was also he who, in 1804, *discovered* the twenty-year-old Bessel and, until his death in 1840, being 27 years older, fatherly took to heart his development and was deeply concerned about his life. And it was also Olbers whose mediation brought about the correspondence between Gauss and Bessel.

They only differed in age by seven years and were delighted by each other. This feeling strengthened even more during the next meeting of Bessel, Gauss, Schumacher and Olbers on 2 Sept. 1809 in Lilienthal. In 1808 Gauss and Olbers rescued Bessel from a threatened conscription. Then, being a professor in Königsberg without a doctorate, Bessel had initially experienced difficulties and Gauss ensured his receiving the doctor degree in Göttingen. Understandably, Bessel appreciated this friendship. *If it will only be possible, dearest Gauss, to prove to you once more how gladly I will be in use for you*, he wrote in his letter of 10 March 1811.

We should think that the foundation of a strong, lasting and unshakeable friendship had thus been laid the more so since the proof of Gauss' deep respect and affection for Bessel was not lacking (*semper totusque tuus* [invariably totally yours]). However, it occurred otherwise. In June and July 1819 Gauss and Bessel many times failed

to meet either in Göttingen or Lauenburg. On the other hand, Bessel's new meeting with Schumacher in Lauenburg in August 1819 became the base for a close friendly connection as witnessed by their correspondence (535 letters from Bessel and 596 from Schumacher).

It became noticeable, however, that over the years the tone of the letters between Gauss and Bessel became less warm and indeed stiff and formal and less was written. About 2/3 of the letters was exchanged in the first half of the duration of their correspondence, and only 1/3, in the other half. On 31 Dec. 1831 Gauss made known the death of his second wife, and his letter still breathed the previous warmth:

For a very long time, my dear Bessel, I have not written you. You have favoured me with two of your priceless works, whereas I, as I believe, have not yet thanked you for the first one. I feel ashamed of my guilt although I am sure that you will forgive me and that even for a moment you could not have thought that I had forgotten to appreciate your scientific communications or the expression of your friendly cast of mind. You certainly know how high, how very high I set them both.

However, for a year and a half your poor friend has been a victim of heaviest domestic sorrows. The outcome of one of them you will easily guess by the colour of my applied signet ring⁴ applied now for four months. The other one, if at all possible, is still sharper and I hardly foresee any end of it apart of my own. But let me be silent about it. These circumstances agonizingly influenced all my scientific activities and brought almost to a complete standstill my correspondence.

Bessel did not condole. That was one of his peculiar features: he was unable to write letters experiencing sympathy. He did not want to seem too soft-hearted and was rather not ashamed of looking cold, as Repsold had aptly remarked. However, in 1840, after the tragic death of Bessel's only 26-year-old son, Gauss did not find a word of compassion either. Between July 1833 and February 1839, for a full 5½ years, Gauss had been completely silent. And from Nov. 1842 onward Bessel changed the previously friendly address to *Highly respected Sir (Herr) and friend*.

What was the decisive reason for such cooling which became apparent not only in the addressing and rarity of letters, but in the very form of communication?

[3] At first, Gauss. On 23 Dec. 1848 he declared that he would have wished to delete the address *Herr* (Sir) from the envisaged but not then brought about publication of Bessel's letters to him, and thus to give no occasion for any assumptions. He, Gauss, was sure that it was not he who initiated that formality. Gauss continued:

I reluctantly mention one more letter which I would rather completely withdraw [the letter of 28 May 1837, see below – K.-R. B.]. At that time it seriously offended me and even still more by its improper tone rather than by the matter itself. Actually in its last lines the tone of Bessel's letter became unacceptable. I never allow myself such a tome with respect to a subordinate. [...] As far as I remember, I did not answer that letter at all.

Gauss went on to report that in a letter of 28 Febr. 1839 he expressed his *resentment* over Bessel's expressions after which Bessel, on 28 June, unsuccessfully, as Gauss understood it, attempted to justify himself. However, Gauss felt that it was a new groundless attack and that that second letter should also be withdrawn.

Here are the appropriate passages.

1. Bessel to Gauss. Königsberg, 28 May 1837

I have read with great interest your electromagnetic investigation published in the previous volume of Schumacher's [Astron.] Jahrbucher [Tübingen, 1836 – 1844]. It was very important for me not only because of the certain and firm advance that will now follow but also since it corrected my false opinion.

I have previously thought that you will turn your attention to the generality and simultaneity of the change in the terrestrial magnetism. Now I see that an exhaustive theory of the entire phenomenon of magnetism and its connection with electricity will be achieved. I can only wish that I will also see its significance set in bright light once you report to us your appropriate studies.

Although little am I justified to hope that my wish has some weight, I will not keep silent that it is only directed to my becoming acquainted with your present occupations as soon as possible. You have never recognized the obligation to promote the present knowledge of things by early reports about a certain part of your studies. You are living for the posterity which is completely contrary to my views. I believe that the more certain will become the fate of your still unpublished results the more completely the rights of the present time are recognized. When the new planets caused your Theoria motus, you had worked not only for the posterity to which your Disquisitiones [arithmeticae] of 1801 also testify.

You would have surely never seen their success which you could have hoped for according to your own yardstick. However, you could not have remained ignorant how intense have the efforts been when trying to follow the path which you had outlined. Where would have the mathematical sciences find themselves not only in your place, but in Europe in general, had you expressed everything which you could have communicated! It is not necessary to continue the less so since I fear to repeat something which you were told a hundred times.

It was this last sentence that had especially hurt Gauss' feelings.

2. Gauss to Bessel. 28 Febr. 1839

I tend to fear that you [...] have been led to a wrong opinion about the aim which I wished to achieve by my work on the general theory of terrestrial magnetism. It is flattering that you appreciate that hasty publication⁵ but I ought to complain about your sharp expression which you used. Only those may be reprimanded for delays who withhold something quite ready, i. e., ready for publication, if only publication is in their power. This is something which I never yet did in my life.

It is a double entendre for a manuscript to be either ready in a fair copy or prepared for publication. For the latter I need time, much time, much more than you can probably imagine since I can only work slowly whereas my time is in many ways restricted, very restricted.

And I also need to be in high spirits (need it much more for preparation in this sense than for the first try) which is regrettably too seriously and in too many ways overshadowed.

And I would therefore allow myself to ask you to judge me more justly.

Bessel had not directly used the word *delay*. In a letter to Schumacher of 23 Dec. 1848 mentioned above, Gauss surmised that there was one more letter from Bessel which he had not kept. However, it is conceivable that during the discussion Gauss only had in mind the *sense* of Bessel's reproaches.

3. Bessel to Gauss. 28 June 1839

At first, allow me to say that I did not mean delays in an unpleasant sense. I have really never even thought that you had wished to conceal the treasury of your ideas from the others. I myself have rather considered this matter exactly as you have explained it. And sufficiently often I had the occasion to be amazed by the maximal thoroughness with which you describe and form your contributions. But I have also understood that such maturity is not compatible with a quick succession of announcing [your results]. Allow me therefore to say something in my defence.

I do not dare to be insensitive either to the significance which a scientific investigation obtains by becoming fully ripe. However, when the time [of work] increases by quantities of the first order, the [studied] magnitudes tend to their maximal [ripeness] by those of the second order. Will not the main idea itself appearing in a respectable, even if not in a maximally attainable description, more rapidly promote science than your postponement until the time favourable for the appearance of your highest degree of quality? Would have Euler achieved as much as he did had he published only a tenth of the great number of his ideas in an irreproachable form rather than the whole lot of them? Would have Lagrange acted better had he left his earliest writings (published in the Miscellanea Taurinensia) unknown for 20 or 30 years until they became completely ripe?

I know that these questions belong to those which could be answered not unconditionally but only according to one or another point of view but I keep to the view from which follows your approval (?). You look disinterested when something is taken away from your great treasure, and really have only to follow your own views, but you cannot be ignorant about the danger of complete loss to which you expose everything that will not be taken away from you.

You can only await that in general all the friends of exact sciences balance their own benefit against your viewpoint. And in any case your contemporaries have even more grounds for wishing that you will be less rigorous. However, I came too far; I only thought of justifying myself.

We can certainly understand that a man as sure of his value as Gauss was, regarded such wishes as expressed by Bessel on 28 May 1837, and moreover formulated in a way smacking of a sermon, like an unbidden tutelage and a wrong assessment of the mode of his studies and publications. Among friends, however, such division of opinion should be allowed.

[4] Commentators have described Bessel as at times somewhat hot-tempered but, in contacts, a jovial, cheerful, brisk and sincere man. Often he went too far. His known argument with the Berlin astronomer J. F. Encke, a student of Gauss, who, due to Bessel, became Director of the Berlin observatory, shows that Bessel could have been implacable. Because of that quarrel even the friendship with his intimate friend, Schumacher, hung by a thread. However, as a rule he changed his tone and attempted to rectify *that which he (?) had spoiled while being in extremely ill humour* (his letter to Schumacher of 16 March 1838), and he did not shy away from expressing afterwards that the matter *really sorrows* him. And in his letter to Gauss of 28 June 1839 he had also changed his tone which certainly did not find in Gauss any requited love for his arguments.

We ought to mention as well that Bessel with his opinion was not at all alone. On 25 Jan. 1825 Olbers, who did not even slightly differ from Gauss, wrote to Bessel:

I am very curious about your discourse on the perturbations of the planets. It can easily happen that you will once more clash with Gauss as it really occurred about the determination of the bending of the instrument. About three weeks before Schumacher had received your letter, Gauss had orally explained it to me.

This is only a coincidence but otherwise our Gauss is often guilty himself when others overtake him with discoveries which he had also found. I am unable to praise you sufficiently, my dear friend, and many of my correspondents thankfully and admiringly recognize that out of sheer love of science you at once make known a new method, a new solution, a new and more correct viewpoint as soon as your genius and your studies discover it without taking into account that others will thus achieve things, solve problems, calculate that which had been too difficult or indeed impossible for them previous to your communications.

It seems however that Gauss invariably wishes to be the first to pick the best fruit to which his discovered and paved path led him before showing them to others. I think that this is a slight weakness of a man otherwise so great, the less to be explained since he had favoured us with so much from his inexhaustible riches⁶.

Schumacher had also conveyed various similar thoughts to Gauss, certainly in his own tactful form, without offending him. Many years earlier Bessel had made known to Gauss his opinion shared by many others that a Gauss can spend his time more usefully than on geodetic work which can be fulfilled by less important people. Gauss answered him on 14 March 1824:

In many letters you have so strongly declared that the results of measurements are insignificant, and thus to a certain extent reproached me for wasting my time and wished me that its loss ends. God almighty, how wrongly you are judging me. However, it is much more important for me not to be falsely judged by you than my reluctance to justify myself.

To tell the truth, I agree with you about the matter itself. All the measurements taken worldwide do not offset a theorem which leads science really nearer to eternal truths. However, you ought to

compare the relative rather than the absolute values. The measurements undoubtedly possess the former. They should connect my system of triangles with that of Krayenhof and thus with the French and the English triangulations⁷. And, however unimportant is their value according to your estimation, to my eye it is higher than the value of the occupations which had been abandoned because of those measurements.

Here (?), I am so far from being the boss of my time. I ought to share it between reading lectures (for which I have long since had an aversion, although not caused, but strengthened by the feeling which invariably accompanies me that I am thus losing my time) and work in practical astronomy. This latter always caused me so much pleasure, that you should admit that when any real help is lacking for the immeasurable number of small and smallest duties, the feeling of loss of time is removed if only you are certain of pursuing a considerable and important goal.

However, for us, others, you made that difficult since you have overtaken us and done away with most of the desiderata in such a masterfully way that for us, others, only little is left aside by gleaning the remains.

So what is left me apart from fleeting hours for a work which I myself can highly appreciate? A person with another character, less susceptible to unpleasant impressions, or I myself if much will change, will perhaps secure more fleeting hours than I generally can. Under the circumstances I do not dare abandon an enterprise, although involving a thousand difficulties and perhaps exhausting my strength, since it is really useful. Someone else can certainly fulfil it whereas I will then under more favourable circumstances achieve something better. However, if I do not take that enterprise on myself, nothing at all will be done⁸. And I dare not conceal from you a fact that to some extent equalizes the disparity between my earnings which remain the same in 1824 as those fixed for you in 1810 by Jérôme⁹, and the necessities of a numerous family.

This explanation did not bear much fruit. On 15 Jan. 1832 a surprised Bessel wrote Schumacher that

Gauss is so occupied with physics while having such a great mathematical treasure in stock. However, only in this respect do I find it so unusual.

Many contemporaries, for example von Humboldt, had also been surprised.

[5] In the abovementioned letter of 14 March 1824 written by Gauss we find no trace of an offence, so why fifteen years later had he reacted so sensitively? Is it sufficient to explain this change by a decrease in the readiness to conciliation with age? I do not think so. I rather perceive Gauss' later reaction in that he, since the dying away of his second wife and the emergence of the known difficulties with his sons by that second marriage (he alluded to this in his letter of 31 Dec. 1831 quoted above), he sometimes suffered from depression whose signs had been felt previously. He therefore became

A queer sort of a fellow [written by Schumacher in English – O. S.] and somewhat more of an egoist than necessary for a pleasant

*contact, but at the same time he is exceptionally honest and incapable of any mean slyness or evasion*¹⁰ (as Schumacher wrote to Bessel).

There are many instances of Gauss' changeable mood. For example, when he stated on occasion that he will not deal with a manuscript sent him since it was badly written. Actually, it was written clearly. This is a proof thereof just as the following description of Schumacher's visit to Gauss in May 1834, see his letter to Bessel of 30 May 1834; however, I am leaving out a drastic comparison¹¹. Gauss, as it emerges, had time and time again expressed what he actually felt about Schumacher:

1. [I] *went at first not to him, but to Bessel.*

2. *With you (with Bessel I) lived fourteen, but with him only a few days.*

3. *Since you [Bessel] had still stayed [in Berlin], I have postponed my departure for a few days. (I cannot guess how he found it out if Encke had not written to him).*

4. *I left him [Gauss] during daytime (not at night) since I thought of living in a hotel rather than in his place. I had written to him from Berlin and asked whether he will allow me to live in a hotel since everyday life in his place was thrown into disarray by the death of his wife. [...]*

But enough of it all! Gauss is certainly unhappy about his dissatisfaction with everything in the world and exactly for this reason anyone who associates with him ought not to take amiss if his foul mood sometimes blazes up like a kindling.

In his answer of 4 July 1834 Bessel called the description above *curious* and continued:

But it follows that our friend is a crass egoist. How else can his foul mood occasioned by a random occurrence which he does not like show up in such a way that your statement [Bessel repeats Schumacher's allegory – K.-R. B.] could have been to a certain extent confirmed.

In 1842, when Bessel himself visited Gauss on his way to England, he was met not better [than Schumacher]. They did not see each other since 1825 when they had spent together only an hour in Rothenburg, on the post road to Bremen. It did not then come to the conversation desired by Gauss since many other astronomers were also present. It should be assumed that this time they will seize the opportunity to continue their previous talk and to rectify mutual offences. Nothing of the sort had however occurred. On 21 Nov. 1842 Bessel informed his friend Schumacher:

You know that I have spent a few days for making a detour and putting up at Göttingen. After having a meal and dressing myself up, I went to Gauss but found him caustic. He spoke about [my] living a while in England and described the diet [there] as pernicious. I thought that I will have to adjust somehow [my meals in England]. I thought of having some soup and a beefsteak for breakfast and doing without regular dinners. When I mentioned the beefsteak Gauss began speaking about teeth exactly the same way as you have written me, so that I did not remain in doubt about the source of his remarks.

It seemed very funny, but fine since otherwise I would have scarcely

got the better of a temptation to remark soothingly about the defect of his own teeth that biting is still enjoyable even if little is achieved. [...] Next morning, however, he was quite amiable so that finally I thought it was nice to have come to Göttingen.

Nevertheless, on 29 Nov. Schumacher took exception: he had corresponded with Gauss about a denture for approximately a year, but not anymore. And on 5 Dec. Bessel reassured him:

Leave both my and his teeth alone, they are not important. Gauss' foul mood must show somehow! I am very far from being upset by foul mood and I only related to you the curious way in which Gauss takes notice of the attention paid him because of that curiosity. He had previously responded exactly in the same manner to your similar attention. Incidentally, I myself am not invariably in high spirits.

Schumacher answered on 21 Dec.:

I had not intended to excuse Gauss' foul mood, I only wished to show that I probably did not directly give him the arrow which he shot at you. He is the most unusual person in the world with whom, in spite of all his rough edges, you cannot really be angry. Attention, as you remark, and as I myself know by my own repeated experience, is usually met with an expression of foul mood. And I therefore find that it is much better just to remain exactly within the boundaries of usual politeness.

You certainly did not know that your travelling through Göttingen was already an attention. And by your very presence you have transgressed those boundaries and had to take the consequences. Weber thinks that Gauss' foul mood sets in because of corns from which he suffers in an unusual measure and testifies that when they seriously bother him, he becomes as irritable and as angry as possible, but that in a few hours, when the pain disappears, he becomes amiability itself. I know by my own experience, that Gauss can indeed be amiable, although not often.

On 26 Dec. Bessel returned to this topic:

There is nothing to say about Gauss. A bit of foul mood is of no consequence. It can be completely forgotten even if it did not entirely disappear the following days. With a head so heavy and sickly legs, how can stable equilibrium always remain?

In spite of this amusing assurance ringing with truth, Bessel, as it seems, had not forgiven the initial unfriendly reception by Gauss. Indeed, his first letter to Gauss after that visit began with a dissociating form of address: *Highly respected Sir (Herr) and friend*. We are led to a suspicion that Bessel had harboured a grudge which was not rectified at their meeting. No wonder that we find in his letter to Schumacher of 30 April 1840 the following text:

I wish to confess gladly that at the time, when my astronomical troubled life had started at Königsberg, and I had resolutely thought of beginning something important, one single approving word from Gauss would have greatly encouraged me. I regarded the abstention from sending me such a word as more than a chance inattention. Those, however, are bygone times and Gauss had won great claims on general respect. In comparison, my own did not last¹².

Gauss acquired a great claim to general respect, and in comparison

with him my own claim has recently disappeared without trace as it should have done. It seems to me that this is a really remarkable statement. We recall the lack of a public recognition of the *absolute geometry* of Johann Bolyai so passionately wished for by his father, Wolfgang¹³. It is doubtful that no trace of Bessel's disappointment over his visit with Gauss had remained since his letters to Gauss became formal afterwards. And we may also suppose that Bessel thought that Gauss had not appropriately appreciated in writing the merits of his son in law, Adolph Erman.

[6] One more word ought to be added about Schumacher's quoted statement. He remarked that in his contacts with Gauss he remained *exactly within the boundaries of usual politeness*. This should be denied. In his letters to Gauss he always expressed himself with refined politeness, and, yes, it is often barely possible to gainsay there certain servility. This, however, is hardly a reproach. Apart from his extensive correspondence with the most important contemporary mathematicians, astronomers and natural scientists, Schumacher only left traces in the history of science as the founder and editor of the *Astronomische Nachrichten* (A. N.).

In astronomy his role was similar to that of A. L. Crelle in mathematics as the founder and editor of many years' standing of the *Journal für die reine und angew. Math.* Because of the deficiency in his knowledge and ability of judging, Schumacher was only able to bring the A. N. to a centre of scientific information by the support of his competent friends, of Gauss, Olbers and Bessel in the first place. He himself had wholly understood it. We are pleased to read for example in his letter to Bessel of 19 Aug. 1842 that if he *did something useful for science, it only is [was] as a middleman*.

And he always therefore endeavoured to act pacifically whereas his report about the ill-starred visit with Gauss in May 1835 was a rare event, and his spontaneous letter to Bessel shows how it worried himself sick. Otherwise the tenor of his letters was directed at preserving or establishing peace between those people on whom he depended. On 25 Apr. 1840 he wrote to Bessel quite characteristically:

[I gladly see], *so warm-heartedly see you and Gauss, the two outstanding people, being on intimate terms. Gauss invariably believes that you are underestimating him¹⁴, and I know how he values each favourable word from you.*

When in 1838 Bessel suspected Schumacher of being in cahoots with Encke to plot against him, Schumacher became horrified and turned, literally wringing his hands, to Olbers and Gauss requesting them to intervene. However, Bessel retracted his announced boycott against the A. N. and sent Schumacher a new manuscript. Schumacher's relief was indescribable.

This digression should only emphasize that Schumacher's advice (about contacts with Gauss) *just to remain exactly within the boundaries of usual politeness* and to do nothing else was only theoretical. He himself happily had not followed it. It is true however, that Gauss had not appreciated exaggerated eulogies. Nevertheless, I think that he had wrongly stated, in a letter to Schumacher of 23 Dec. 1848, that many compliments found in almost each letter from [the

late] Bessel to him only reflected his beloved pursuit, *an attempt to say something pleasant to people or something that he believed they will be glad to hear*. And he knew quite well how much should be subtracted from his statement.

[7] We have no reason to suppose that Bessel's expressions of wonder and deep respect for Gauss' mathematical genius were exaggerated or even hypocritical. And in his letters to Schumacher in which, as we saw, he had not minced his words, we also find sufficient proof that he fully appreciated and respected Gauss' *mathematical power*. It could not have been otherwise. For example, when he heard from Poggendorff, to whom he was usually *well disposed*, a wrong appreciation of Gauss' paper on magnetic problems, he informed Schumacher about that on 31 Aug. 1839 and added that he was so angered that *most of all* wished to explain to everyone what had Gauss really meant although this is the business for Gauss himself if only he will consider it necessary.

On 9 Sept. Schumacher found such an intention very honourable:

But you also know what kind of a person he is. He will certainly do nothing. He is satisfied by putting down his works without troubling himself about them. If they are misunderstood, he will laugh inwardly and perhaps only become angry if his works are deliberately corrupted. In conversation he never argues when being in the right, but applies the entire art of dialectics to defend a wrong proposition which he had stated¹⁵.

Other contemporaries sided with Bessel in criticizing Gauss for other matters. In a letter to Schumacher of 5 April 1835 Bessel found Gauss (1825) *excellent, worthy of admiration*, but he added that he did not understand why Gauss had not referred to Lagrange (1779).

Did not he know about the work of Lagrange? I believe in this possibility as little as I do in Gauss' denial of his knowledge had he been asked about it. This, however, is his habit not to be commended of naming no one else.

In a letter to Schumacher of 2 April 1836 Alexander von Humboldt reproached Gauss for the same reason: *Next to the map projections, as a threatening spectre, appears the ghost of Lagrange*. On 8 April 1835 C. G. J. Jacobi, the second after Gauss most eminent German mathematician of those times, wrote Bessel: For Gauss, *de mortuis nil nisi bene* [nothing but good should be spoken of the dead] *is replaced by de mortuis et de vivis nil* [nothing about either the dead or the living]. And on 21 Sept. 1849 he wrote to his brother Moritz about Gauss: *For over twenty years he had not ever quoted either me or D [irichlet – K.-R. B.]*.

Gauss himself had named the reason for his behaviour. In a letter to Schumacher of 6 July 1840 he wrote:

I reluctantly express myself in detail about the achievements attained by others working in the same field as I did, if only not being entirely convinced in that I really may mention them approvingly.

And, again,

Nevertheless, I recognize [...] that I did not at all study critically [the history of the theory of magnetism – K.-R. B.]. [...] As a rule, I am unable to decide just like that who should be favourably mentioned

and thus to reinforce myself unconditionally. And, when desiring to provide authoritative connections, it would have been necessary to conduct [prior] literary studies for which I have neither time nor (I confess) inclination. Indeed, such investigations are not exactly to my taste.

I can only say that the forbidden to a usual author should probably be allowed to a Gauss, and that at least we ought to respect his grounds.

[8] Finally, I would like to mention one more event, important for describing the relations between Gauss and Bessel and at the same time typical for both of them. Gauss had published a paper in Schumacher's *Astron. Jahrbücher* for 1836, Geomagnetism and the Magnetometer although he expressed his aversion to any popularization of scientific achievements. In this respect he was a *scientific aristocrat* through and through, but, to oblige Schumacher, Gauss had overcome his doubts. It occurred, however, that Humboldt had misunderstood various details, and in a letter of 15 April 1836 to Schumacher Gauss resigned himself to ascertaining that in spite of all his efforts, he apparently did not impart the necessary clarity to his paper.

Contrary to Gauss, Bessel gladly provided generally understandable essays on the state of knowledge¹⁶. In the same *Jahrbuch* for 1843 he published a paper (Bessel 1842) on geomagnetism. And then Gauss began to think that Bessel did not at all appreciate his contribution, as Schumacher informed Bessel on 26 Jan. 1843. These are his words:

*I have informed him right away that I know the opposite from your previous statements, but I would like to ask you to send me a few words which I may communicate to our old friend. You yourself surely do not wish him to suffer from false ideas. He concludes them drawing on your paper in the *Jahrbuch* for 1843. He believes that it is as though a reproach to him, as though until now he did not inform the public about the new advances in the teaching of magnetism.*

Nevertheless, you had mentioned his paper of 1836 (which you therefore knew about) but merely since it discusses the connection between galvanism and magnetism. This discussion only occupies a quarter of the paper mentioned so that you ignore or negate the other three quarters. There, he at first based that doctrine, which in all known to him books was described confusedly, and then, which is the most important point, he managed to explain geomagnetism for laymen.

What will you say about these conclusions? I have painfully seen how such a powerful mind can be deluded when it is buttoned-up and alone and only allows sullen ideas to take their course and does not suspect anywhere kindness or friendship. I am far from being able to laugh and will rather comfort and impart friendly views to him. His letter has no trace of anger or morbid vanity and is rather sorrowful since he sees himself misjudged. But it seems that there is nothing with which to oppose him except by showing him that he is not misjudged.

On 6 Febr. 1843 Bessel answered very reservedly since he intended to leave Schumacher the possibility to weaken Gauss' doubts by passing on his letter to others. However, in a supplement he expressed

himself quite angrily:

I do not understand what G[auss] really wishes to say. There is not a single word about the Theory of geomagnetism in his paper of 1836¹. His intention, as it seems, was either for me to rewrite his paper or that I should have chosen to refer to his paper instead of adding a few words with which I explained the [work with the] magnetometer and the determination of the absolute intensity according to the essence of the matter. [...] That I have otherwise deviated from his description – I cannot excuse it anymore. [...] I knew long ago that even with the best will in the world it is possible to act otherwise and wrongly towards others.

There are other proofs of the approach and strengthening over the years of the disagreement between Bessel and Gauss which however never diminished the mutual appreciation of their significance in the field of science. Nevertheless, the above is sufficient for answering the question which is felt between the lines of their letters as quoted above.

Two as distinctly complicated personalities as those of Gauss and Bessel spatially separated from each other, only meeting very seldom and under unfavourable circumstances and finding themselves on those occasions so changed into the bargain that they could have *not recognized one another anymore* (Gauss to Schumacher on 19 June 1842) perhaps inevitably had to become estranged from each other.

In old age Gauss, like Bessel, acquired an inclination which the latter must have thought of having even as a young man since he said: *I begin wishing to suck out poison out of roses.*

Had the intention to invite both Gauss and Bessel to Berlin been put into effect, their relation would have certainly turned out differently, but the finale became all the more peaceable. During Bessel's protracted and agonizing mortal cancerous disease Gauss kept silent, but, having learned about Bessel's death, he wrote to Schumacher on 25 March 1846 that he felt himself

Most painfully shaken, although we had to expect his death and to wish a speedy end of his suffering. Our contacts began in 1804 and now only a few old friends are left. So let us, dear Schumacher, all the more hold together.

Bessel's fundamental philosophy [fundamental appreciation of Gauss] was expressed in his letter to Schumacher: *Gauss is the open and clear truth itself.*

[9] One more remark is perhaps not useless. Here, in the extracts from the letters, there appeared much rarely spoken of but no belittling of either of the two heroes of the mind is perceived there.

Truth is indivisible. Even a Gauss and a Bessel are human which should not be either overstressed or put down. A biographer must also include that which does not coincide with the information transferred to him with an eye on the living and the picture he draws differs from the previous image, cf. the apt remark of Gerardy (1964, esp. p. 6).

We intend and may find out, we portray and judge the individuality and peculiar features, the lasting and the transitory, according to witnesses, be they contemporary or not. Such a portrayal brings Gauss and Bessel humanely nearer to us. Here, we may appeal to Gauss'

own words from his letter to Schumacher of 30 May 1846:

Concerning the topic of your second letter [of 27 May – K.-R. B.] it is not quite clear to me why you are so opposed to the publication of Bessel's correspondence. It certainly contains much information important for science, but posterity will also regard a correspondence that depicts not astronomers, but people as a very valuable legacy. The correspondences of Leibniz, Kepler, Euler, the Commercium Epistolicum¹⁷ and so many other similar collections form a priceless treasury. Only suppress that, which can harm some living people, and it becomes possible to publish all the rest.

Notes (O. S.)

1. Biermann himself (§ 9) quoted Gauss who had mentioned few extremely valuable collections of letters.

2. Cf. Jacobi's statement in § 7.

3. Korn & Korn (1961/1968, § 7.5.1) called that theorem after Cauchy and Goursat.

4. The colour of stamps and possibly signet rings, though perhaps not when used by individuals, had a heraldic meaning. Gauss apparently applied the black colour.

5. Gauss (§ 8) had nevertheless highly valued that *hasty publication*. There also Bessel declared that there (in that paper) was not a single word about the *theory of geomagnetism*. However, in his letter to Bessel of 28 Febr. 1839 (§ 3), Gauss mentioned his work *on the general theory of terrestrial magnetism*.

6. Biermann quoted the last paragraph in his later paper [III, § 5]. Concerning the *clash with Gauss about the determination of the bending of the instrument* I can only add that Gauss determined it in 1828.

In a letter to Gauss of 28 June 1839 Bessel notes that much can be lost with Gauss's death, and it is opportune to add that Gauss himself stated the same in his letter to Bessel of 15 Nov. 1822.

7. Gauss (Nachlass; W-9, pp. 402 – 403) set high store on the unification of the separate triangulations which existed in various parts of Europe.

8. A few lines above Gauss stated the opposite. Another contradiction in the same letter concerns the significance of geodetic measurements: their value is higher than the value of the *occupation which had to be abandoned*, but, instead of these measurements, he could have *achieved something better*. Finally, someone else could have replaced Gauss, but, at the same time, nothing will be done without him.

9. Gauss should have mentioned Johann Schröter instead of the mysterious Jérôme.

10. Biermann quoted the last paragraph in his later paper [II].

11. Biermann's decision is unacceptable since the deleted passage was included in an unpublished source. And in § 9 he himself expressed an opposite opinion.

12. Bessel apparently forgot that Gauss had ensured his receiving the doctor degree and that Gauss and Olbers had rescued him from a threatened conscription, see § 2. Even in 1828 Bessel complained that Gauss had overshadowed him. Then, in 1843 Bessel, in correspondence with Gerling, attempted to establish his priority over Gauss in the adjustment of triangulation and accused Gerling of failing to mention his, Bessel's (non-existing) merits in the development of the theory of probability (Gerling 1861).

13. An explanation is needed. Gauss had not publicly stated his views about that *absolute geometry*. Gauss wrote to Wolfgang Bolyai endorsing the discovery, but he also asserted his own priority, thereby causing the volatile Janos to suspect a conspiracy to steal his ideas (May 1972, p. 302, right column).

14. At the same time (see below) Bessel had repeatedly complimented Gauss, a fact which the latter wrongly interpreted.

15. Biermann quoted the last paragraph in his later paper [II, § 6].

16. I categorically disagree. His popular writings were superficial and contained serious mistakes.

17. *Commercium Epistolicum* published by the Royal Society in 1712 was a collection of letters bearing on the priority strife between Newton and Leibniz.

Biermann quoted the last sentence in his later paper [III, § 3].

Brief Information about Those Mentioned

Auwers Georg Friedrich Julius Arthur von, 1838 – 1915,
astronomer

Bolyai Farkas (Wolfgang), 1775 – 1856, mathematician, a friend of
Gauss

Bolyai Janos, 1802 – 1860, mathematician, one of the discoverer of
the non-euclidean geometry, son of Farkas Bolyai

Erman Georg Adolph, 1806 – 1877, physicist, geophysicist, editor
of the correspondence between Olbers and Bessel

Krayenhoff Cornelis Rudolphus Amandes, 1843 – 1921, general,
physicist, engineer, geodesist

Poggendorff Johann Christian, 1796 – 1877, physicist, the author of
the many-volume *Biographisch-Literarisches Handwörterbuch*,

Schröter Johann Heroymus, 1745 – 1816, astronomer

Schwarz Hermann Amandes, 1843 – 1921, mathematician

Bibliography

Biermann K.-R. (1963), Aus der Vorgeschichte der Aufforderung A. von Humboldt von 1836 an den Präsidenten der Royal Society zur Errichtung geomagnetischer Stationen. *Wiss. Z. Humboldt-Univ. zu Berlin*, Math.-Nat. Reihe 12, pp. 209 – 227.

Briefwechsel Bessel – Schumacher, Bessel – Jacobi. Nachlaß Bessel. Akad. Bibl., Berlin-Brandenburgische Akad. Wiss.

Briefwechsel Weierstraß – Schwarz. Nachlaß Schwarz. Ibidem.

Briefwechsel C. G. Jacobi – M. H. Jacobi (1907). Leipzig.

Bruhns C. (1869), *J. F. Encke*. Leipzig.

Gauss C. F. (1825), Allgemeine Auflösung der Aufgabe: Die Teile einer gegebenen Fläche auf einer andern gegebenen Fläche so abzubilden daß die Abbildung dem Abgebildeten in den kleinsten Teilen ähnlich wird. *W-4*, pp. 189 – 216.

--- (1836), Erdmagnetismus und Magnetometer. *W-5*, pp. 315 – 344.

--- (1880), *Briefwechsel mit Bessel*. *W/Erg-1*; K. E. Henrici. Berlin, Auktion-Kat. 120. 1927, No. 93; J. A. Stargardt. Marburg, Auktion-Kat. 558.1962, No. 579.

Gaede (1885), Beiträge zur Kenntnis von Gauss' praktisch-geodätischen Arbeiten. *Z. f. Vermessungswesen*, Bd. 14, pp. 113 – 137, 145 – 157, 161 – 173, 177 – 207, 225 – 245.

Gerardy Th. (1964), *Mitt. Gauß-Ges. Göttingen*, No. 1.

Gerling Ch. L. (1861), Notiz in Betreff der Prioritätsverhältnisse in Beziehung auf die Methode der kleinsten Quadrate. *Nachr. Georg-Augusts-Univ. und Kgl. Ges. Wiss. Göttingen*, pp. 273 – 275.

Korn G., Korn Theresa M. (1961), *Mathematical Handbook for Scientists and Engineers*. New York, 1968.

Lagrange J. L. (1779), Sur la construction des cartes géographiques. *Nouv. Mém. Acad. Roy. Sci. Berlin*, pp. 161 – 210.

May K. O. (1972), Gauss. *Dict. Scient. Biogr.*, vol. 5, pp. 298 – 315.

Repsold Joh. Ad. (1918), H. C. Schumacher. *Astron. Nachr.*, Bd. 208, 17 – 34.

Sheynin O. (2001), Gauss, Bessel and the adjustment of triangulation. *Hist. Scientiarum*, vol. 11, pp. 168 – 175.

II

The change of our concept of Gauss

Wandlungen unseres Gaussbildes.
Mitt. Gauss-Ges. Göttingen, No. 28, 1991, pp. 3 – 13

[1] I will try to say something about how our picture of Gauss *as a human being* has changed during 31/2 decades since new sources have been discovered and the interpretation of the literature has deepened, and how does it change nowadays (Biermann 1978); [IV]).

The high respect for the *scientific grandeur* of Gauss has not changed at all, and we may just as well say, as Richard Courant did here in Göttingen in 1955 on the occasion of the centenary of Gauss' death:

A hundred years have passed since Gauss' death, but his scientific grandeur remains as mysterious and incomprehensible as it should have been for his contemporaries. The prophetic intuitive originality, the depth and versatility of Gauss' achievements coupled with an incredible display of power and tenacity are unique both in his purely theoretic work and applied fields.

It seems to me that Gauss' mathematical work is somewhat more remote from today due to the rapid development of mathematics and its applications both in means and in form about which Gauss could not have had any notion. However, the wonder as expressed by Courant had not disappeared at all. But how do the matters stand with regard to Gauss *as a human being*, to the topic which interests us now?

In 1955, he had still been inconceivable, unyielding to measurement by human scales. For a century he had remained a marble statue of a hero and emanated a sense of calm and superior composure without any human warmth, required respect. The sculptors of this cold statue belonged to the inner circle of Gauss' surroundings during his last two decades: the physicists Wilhelm Weber and Johann Benedikt Listing, the biologist Rudolf Wagner, the physician Wilhelm Baum, the orientalist and Gauss' son in law Heinrich Ewald, Gauss' youngest daughter Therese who had been keeping his house, but, first and foremost, the mineralogist and geologist and Goethe's godson Wolfgang Sartorius Freiherr von Waltershausen.

The role of the last-mentioned which he played in Gauss' surroundings can be compared with that of Eckermann in the life of Goethe. He carefully recorded the important statements and removed or omitted everything that did not seem to fit the flawless marble statue. Thus he at once became the main source for feeding the biographies of Gauss and will certainly remain as such.

Had a member of the circle mentioned above out of some subjective interest really intended to reveal to the public a human side of that statue, an instantly arisen mighty storm of protest would have quenched any such project. We cannot imagine that Sartorius lied, but neither did he state the whole truth and his omission of everything contrary to the notion of reverence coloured the picture. A few examples will elucidate that conclusion.

Contemporaries certainly knew about the conflict between Gauss and his colleague and former friend Harding, the discoverer of the minor planet Juno, who was appointed extraordinary professor and inspector of the Göttingen observatory two years ahead of Gauss, together with whom Gauss had worked for 27 years. Sartorius did not say a single word about that conflict, whose deeper causes we still do not exactly know, although it strengthened with time and even led Gauss to think about leaving Göttingen.

Just the same, Sartorius kept silent about both the temporarily threatened break of Gauss' engagement to his future second wife and the estrangement between Gauss and Bessel. Then, he presented the establishment of the so-called Göttingen Magnetische Verein (Magnetic Union) as the result of an *encouragement* by both Gauss and Alexander von Humboldt but he passed over in silence an initial considerable ill feeling between them. Indeed, Humboldt realized that in a short time Gauss had mastered both theoretically and practically *that*, which he was accustomed to regard as his own field of knowledge. In addition, Gauss certainly noticed Humboldt's inadequate expertise in his initial opinion about the new Göttingen facilities for observing geomagnetic phenomena.

Then, Sartorius completely left out Gauss' strong clashes with his elder son by the second marriage and his serious worries about the future of his second son by the same marriage, his ensuing excitement and fears which for a long time poisoned his life. Sartorius affirmed that Gauss' faith was *unshakeable*, but his conversations with Wagner show that Gauss *envied* those who were able to believe *right off the heart* and asked: *Tell me, how to begin.*

Sartorius testified that Gauss *could have doubtlessly become an excellent finance minister*, but did not justify his statement. In any case, having been indigent and frugally paid (to compare: Humboldt earned six times more), he made about 500,000 marks by buying and selling securities. By our present yardstick, and taking into account the purchasing power at those times, at the moment of death he thus became a millionaire.

In Göttingen, his skill in increasing his fortune was almost proverbial. This is known from Moritz Cantor, a historian of mathematics, who attended a Gauss lecture together with Dedekind in the winter term of 1850/1851¹. For Sartorius such communications were too banal. He concluded that Gauss was an enemy of any miserableness and naturally overlooked a fact discovered in 1977 by my late friend, Dr Theo Gerardy, an honorary member of the Gauss-Gesellschaft: until an objection was raised after a financial check, Gauss had regularly paid his dues in connection with arc measurements and geodetic work in usual low-value coins but received his payments in gold². From 1825 to 1827 he thus gained 230 thalers.

[2] By leaving out everything which Gauss showed as a human being with his contradictions, doubts and attempts, not free from his moods, sufferings and struggles, Sartorius erected a monument to an *iron* Gauss which for a hundred years decisively determined the judgement of the posterity.

We also ought to consider that, during his later years in Göttingen, Gauss was high above his circle of friends. Witness Wagner:

My friends and acquaintances will attest that we never regarded our great mathematician as a colleague, but always as a superior endowed with wholly unusual spiritual power before whom one always stepped a few paces aside. I will not be misunderstood if I say that in our scientific republic he played about the same role as the lion in the animal fabled world.

For his part, Sartorius tells us:

We never saw a man with a more impressive outward appearance. All the other ones seemed on a par with us, but he stood as an unearthly being, as a priest at his post by the throne of the Deity.

These descriptions are surely somewhat prettified. However, in 1976 the surely highly impressive Gauss was turned into *an aging, shorty and somewhat stout man, a very German professor with many prejudices*. Then, in 1954 the conditions of his life were described as *meagre* with him only getting rid of *oppressive money troubles at a venerable age*. These descriptions are just delusions. I do not dwell on them since they had not practically speaking influenced our portrayal of Gauss.

But it should be remarked that it was not only his Göttingen circle of friends who recognized Gauss' absolutely special position. There rather existed only quite a small number of eminent scientists in the history of science who, like Gauss, enjoyed recognition as such by their contemporaneous professional colleagues. Olbers affirmed that, had Gauss managed to come to Paris, he would be received better than *any scientist was until now*³. In Napoleon's outer office Lindenau heard that only Gauss would have been named a successor to Lagrange.

And Lagrange himself stated that the young Gauss had *by a single spurt raised himself to the rank of best mathematicians*. Laplace talked about *an unearthly spirit in a human body* and Bessel cried out: *What a day is coming, so Sie es wollen!* [just as you wished!]

In the letters of his learned friends we find a wealth of expressions of wonder at the *incomparable genius*, at his *not yet attained* [by others] *perfection*, at the *mathematical giant*. It was believed that he seemed to belong to *superior beings*, he was called *the master* of all professional colleagues. Already in 1804 Humboldt stated that only *one person* called Gauss can impart new lustre to the Berlin Academy⁴.

The King of Hannover approved the legend on a medal commemorating Gauss that called him Princeps, Prince of mathematicians. These words were not recommended by a competent counsellor; quite simply, they were in keeping with the general conviction¹.

So how can we blame those who, due to the superiority of the revered man, are hindered from decreasing the distance to him, and wish to delete everything from the image of their hero that according to *their yardstick* can darken it? However, and we should not overlook this circumstance, the glasses through which we see Gauss have been manufactured not only by Sartorius and the circle of Gauss' friends,

but also by Gauss himself. Conscientiously or otherwise, he powerfully assisted them, and I will prove it.

[3] In the second part of the biography of Gauss written by Sartorius, we find numerous verbal expressions exactly repeated in Gauss' letters, and here are a few examples.

As stated by Sartorius,

Although Gauss perhaps trusted analytical calculus more than any living person, he was considerably ill-disposed towards its mechanical applications of any kind. He attempted to restrict its use as much as it was possible under the circumstances. He often told us that he never takes up his pen for calculations until he completely solves the problem mentally. For him, analytical calculus only appeared as an aid which he uses when completing the task.

On 31 Dec. 1831 Gauss wrote Olbers about the same topic:

For geometers of the first rank calculus always is just the clothes in which they show what they had obtained not by its help but by meditation on the essence of the matter.

And in a letter to Schumacher:

I require that by each application of the calculus or notions one should always retain in mind the initial conditions and never consider the outcome [alle Produkte des Mechanismus] exceeding his obvious right as his own property.

Another example is provided by statements about the *antieclidean geometry* as recalled by Sartorius on the one hand and by Gauss' letters to Gerling and Bessel on the other hand. Significant is also the coincidence of the report made by Sartorius about Gauss' interest in the mortality of babies and old men and his letter to Humboldt which discusses the same subject in about the same wording⁵. It is highly unlikely that Sartorius had seen that letter (only published in 1965) and neither did Gauss compile any summaries. This astonishing coincidence can only be explained when assuming that Gauss had insistently expressed the same thoughts in conversation and in the letter and that Sartorius made very conscientious notes.

Other cases of such facts can be provided, for example the coincidence of the motives for Gauss' study of the Russian language in his letter to Schumacher of 17 Aug. 1838 and in a communication by Sartorius. Or take the statement of Sartorius, *The thirst for truth coupled with a sacred drive for fairness mostly describe Gauss' noble nature*, and, on the other hand, the letter to Steinheil of 16 March 1836: *My theory is certainly dear to me, but infinitely dearer is the truth.*

[4] The often amazing statements more of which can be provided leave no other possible conclusion except believing that Gauss had verbally *and* in writing repeated definite maxims and reflections which especially captured his imagination to ensure their dissemination. I have long ago become convinced in that Gauss wrote and spoke for posterity just as, for example, Goethe and Wilhelm von Humboldt did. Understandably, he regarded these utterances as *publications*. With justified self-confidence he wrote to his publisher Perthes⁶ about his *Theoria motus*: *It will be studied in [a few] centuries as well.* And not without pride he remarked to Sartorius

about his *Disquisitione arithmeticae* of 1801 that *it belongs to history*.

And when drafting his letters he thought not only about the recipients but about his future readers as well and almost always checked himself. We can infer this from his letter to his intimate friend Schumacher when this latter corresponded with him about the envisaged but only 35 years later brought about publication of his correspondence with Bessel. On 30 May 1846 Gauss explained:

Only suppress that which can harm some living people, and it becomes possible to publish all the rest so far as it is of some interest.

However, he wished to omit *one* letter from Bessel. It was exactly such that did not fit the picture of himself which Gauss intended to sketch and preserve. It concerned the question as to whether Gauss reasonably published only quite perfect materials or whether he could have greater contributed to the development of mathematics by lowering his requirements for the preparedness of manuscripts and publishing ideas even when they were not fully ripe and thus stimulating and supporting contemporaries.

Gauss became most highly annoyed and wished to omit the whole letter. Exactly this case can prove how Gauss himself took care of propagandizing his point of view. I am also thinking about letters which he wrote to Schumacher and his previous student, Encke. There he attempted to explain insistently not only them, but later readers as well, why does he hate overhasty publications and only intends to make known ripe materials, is not prepared to provide building blocks but prefers to erect finished structures. Such work requires *very* much unappreciated time, and he puts up with the ensuing delays and the danger that others will overtake him or that much will be lost after his death. His motto is: provide something perfect in essence by tying together the derived insights – or nothing at all⁷.

Even much earlier Gauss had considered it very important to clarify his point of view. Indeed, in September 1814, during a trip to Seeberg near Gotha, he spoke to Encke who recalled that conversation more than 20 years later:

You had then explained your method of work and therefore did not approve of Euler's attitude. He published the results of his reflections perhaps just as they had first presented themselves and only remarked that he will repeatedly and often return to them. On the contrary, you always intend to attain perfection and intrinsic satisfaction both in essence and form.

It is therefore understandable how offensive was for him Bessel's rhetoric question of 28 June 1839:

*Would have Euler achieved as much as he did had he published only a tenth of the great number of his ideas in an irreproachable form rather than the whole lot of them?*⁸

It is not necessary to stress that Sartorius precisely described Gauss' efforts to attain perfection as he formulated it in his letters.

[5] I summarize: Gauss knew that Sartorius, not being a mathematician, will record his main statements and was convinced that his letters will sometime be published. He therefore took care that, by laying almost the usual stress in conversation and insistently emphasizing in his letters those principles will be preserved which he

thought undoubtedly deserving to be saved. And he himself thus assisted in sculpting that statue which had been appearing to the amazed posterity for about a hundred years and presented an idealized hero, a *mysterious and incomprehensible* superman rather than a human being.

After those hundred years clear signs of a change of our perception of Gauss became visible. It occurred that an *iron* Gauss, a *bronze block* are out of the question. Gauss was rather extremely sensitive, influenced by his moods, doubting, seeking, not rarely suffering, sometimes however cheerful man (which Sartorius had not passed over in silence).

To be sure, some documents which had been known earlier did not quite well fit Sartorius' frames and, moreover, they were written by Gauss' most intimate friends. I recall for example a remark made by Olbers in his letter to Bessel of 25 Jan. 1825 about the abovementioned restrictions on the publication of letters. Gauss highly appreciated him both as an astronomer and a human being, and, for his part, Olbers had many times showed his high opinion of Gauss by many deeds rather than words. And this is what he wrote:

It seems that Gauss invariably wishes to be the first to pick the best fruit to which his discovered and paved path led him before showing them to others. I think that this is a slight weakness of a man otherwise so great, the less to be explained since he had favoured us with so much from his inexhaustible riches in ideas.

Such mostly affectionate criticisms which reappear on occasion or other testimonies, for example, Gauss' startling weep over his first wife, some letters to his sons by the second marriage which became known were simply disregarded or remained ineffective since they did not fit the Sartorius' picture. But in any case such facts multiplied until about 1955 a change had emerged. In such cases it is always difficult to set an exact date. I believe, however, that the beginnings of the great and not yet completed change can be fixed at about that mentioned year, 1955. It was then that two editions of the renowned biography of Gauss (Worbs 1955) had appeared. There, for the first time, his depression was discussed⁹ and a note hidden in his mathematical records was published: *Death is preferably for me to such a life*. It was like a thunderbolt destroying an idyll, it was simply impossible to reconcile such a change with the then current picture of an unshakeable Gauss.

I am unable to present here in detail the new aspects discovered in the investigations concerning Gauss since 1955, and I restrict my description to some main points.

[6] The appearance of supplements to the previously published correspondence of Gauss with Gerling and Schumacher allowed the abovementioned Theo Gerardy to illuminate clearly a chapter in the life of Gauss which had previously remained largely in the dark. He described the disturbed relations of Gauss with both sons by the second marriage.

I have already said that even earlier some published documents made some conclusions possible but that these were not used. Gerardy showed how Gauss, helpless and confused, had to apply to his friends

from other cities to settle problems which only properly concerned his family. How, first of all, his former student, the physicist Gerling from Marburg, weakened the panic by a sober and objective consolation, qualified the significance of the problem by life experience and showed the way to solve it by practical advice.

This way was indeed chosen and both sons acquired a possibility to prove their worth in the USA, to show that they are not at all *lost*, as Gauss initially thought at least about the elder son, Eugen. He was undoubtedly the most gifted of all Gauss' sons and the only one who inherited his father's visual perception of numbers.

Theo Gerardy quite justifiably summarized:

The relations between Gauss and his sons show a picture somewhat different from the heroic image which is described in his biographies. Only externally he is unshakeable and unapproachable; actually, however, easily hurt and then virtually helpless. Except [the possible case of] rapid and clear decisions, in such situations which only properly concerned the parents, he has to ask advice from his friends. He cannot take advantage of his position for paving the way for his sons, he was loath to soliciting. He treated them justly and thoughtfully, but we can seriously doubt whether he loved them as much as his daughters. His thoughtfulness, fanatical striving for truth and [possible] opinions of the outsiders from his social surroundings could have robbed him of sympathy for the humanly forgivable weaknesses of his uncontrollable but in essence worthy and kindred son Eugen¹⁰.

Another essential cause for modifying our understanding of Gauss was the finding and use of the relevant correspondence of his friends about which I have reported [ii]. The letters gained the access to an almost unknown previously side of his nature: on the dependence on his moods. In his own letters, he often stressed his need for both cheerfulness and steadiness in his relations with others and judgements. He essentially depended on his mood which in turn was determined by external circumstances, but this was not taken into account in spite of his occasional statements that *We govern over our actions but not over the effects of life conditions on our soul*.

Family discord and illnesses, the need to decide his future, the appearance of sudden events, all kinds of deadline pressure, the duty to read lectures to ungifted students, hot or stuffy weather, – all this unfavourably acted on his mood. It cannot be doubted anymore that Gauss was a person influenced by circumstances rather than a hero, untouchable and existing above the everyday life as described by Sartorius.

Schumacher, probably his most trusted correspondent, knew well enough that association with Gauss without acquiring a foul mood was only possible for those who were able to remain exactly within the boundaries of usual politeness. And Schumacher came to understand that Gauss is *A queer sort of a fellow* [written by Schumacher in English – O. S.] *and somewhat more of an egoist than necessary for a pleasant contact, but at the same time he is exceptionally honest and incapable of any mean slyness or evasion*.

Gauss can be kindness itself, *although not often. In conversation he*

never argues when being in the right, but applies the entire art of dialectics to defend a wrong proposition which he had stated.

I stress once more: all this comes not from someone of whom Gauss had disapproved, but from Schumacher, from a man whom he deeply respected and who remained as near to Gauss as hardly any other contemporary. Schumacher communicated his judgement to Bessel who confirmed it by his own experience. And the last conversations of Gauss with Rudolf Wagner published in 1975 was a step in the direction of a new portrayal of the former. They show Gauss as a weak man who had to attempt to keep cool under affected calmness shown to the outer world.

When the picture of Gauss is thus corrected, much of what seemed mysterious becomes clear. For example, *the contradiction* between the startling weep over the loss of his first wife and the new marriage contracted only ten months later. *The contradiction* between his melancholy mood after the death of his second wife and the disappointment over Eugen and the picture provided by the sister of Wilhelm Weber shortly afterwards. She described Gauss as a cheerful and almost lively person.

Justified become the words of Alexander von Humboldt, *For a free and agile nature like that [of Eugen] coexistence with Gauss was not as easy as desired. The contradiction* between Humboldt's judgement about Gauss as an *intolerantly sensitive*, and a *scientific despot* and, on the other hand, as a *fully warm-hearted softie*. Or *the contradiction* between the feeling for fairness and, as Jacobi once overstated, Gauss' habit of saying nothing about either the dead or the living¹¹.

[7] Allow me to mention two more inclinations of Gauss upon which new light has been thrown during recent decades. I bear in mind Gauss' tendency towards encoding both the achieved conclusions and minor matters, and on the other hand towards recording numerical results even of non-scientific origin. Both inclinations sometimes manifested themselves at the same time.

Thus, the probability that some outsider in Braunschweig was able to gain an insight into the number-theoretic findings of the young Gauss was practically zero and it was just as low concerning the significance of the count of steps from Braunschweig to Helmstedt (once Gauss counted 45,053 of them). The only reason for encoding both events was Gauss' pleasure in his game [with numbers]. He was a *homo ludens*, a playing man delighted by even useless games with numbers, delighted *to act as though* someone was hunting for his newest discoveries and as though he ought to prevent their efforts by encoding.

Being 25 years old, Gauss himself, in a letter to Franz von Zach, admitted that he was a *lusus ingenii* (an inborn player) and 45 years later he wrote to his intimate friend Schumacher:

In general, I am lenient with imagined games. [...] No, I do not deny that I sometimes amuse myself in a similar way but I will never publish anything of that kind.

When Gauss recorded in how many thousands of numbers he had counted the number of primes during a day, he encoded not only the result, but also the relevant dates so that it certainly was an amusing

game. When he also noted the number of weekdays on which he counted those primes and encoded those days by numbers 0, 1, ..., 5, 6, it was a game just as well. In addition, he assigned number 1 to Wednesday possibly because he first saw the light of day on a Wednesday.

Allow me to insert a word about how I became able to decipher the encoded dates. It was known that Gauss congratulated Humboldt on his 30,766 day of life, or at the age at which Newton had ended his *terrestrial career*. I knew that Gauss could have represented dates by numbers, so I began to check whether they can conceal a date. Soon I struck gold. I came across the number 7219 and established that the date on which Gauss defended his dissertation, 16 July 1799, was exactly 7291 days after his birth. Any residual doubt has therefore disappeared: Gauss had indicated number 7291 or 99-VII-16 and added the letter D (doctor).

Soon afterwards Dr. Gerardy sent me a reproduction of a handwritten table which Gauss had inserted in a table of logarithms and called it *Count of days*. For non-encoded numbers, such as 1777 April 30 (Gauss' birthday) this means that, for example, $64768 = 4$ Wednesday (here, Wednesday was not denoted by 1 so that Gauss was not consistent in such things¹²).

So here was a Rosette stone of sorts, a bilingual concordance. The table provided the number of days which passed from Newton's birth to given dates. I am sure that many numbers in Gauss' posthumous manuscripts which have nothing in common with the mathematical contexts are actually dates. Thrifty Gauss put them into printed texts and thus economized on the relatively expensive writing paper. Why did he indicate this or that date is certainly not easy to determine, and furthermore it will be necessary to establish on which weekday their count began with a zero.

I cannot here describe the decoding of *combinations of letters* and restrict my account by referring to my relevant contributions¹³ in which I had indicated in detail Gauss' pleasure of encoding. However, I would like to present a typical example of his table of numerical results, a reproduction of a page from Gauss' *Mathematical Diary* [omitted in translation], Nieders. Staats- und Univ.-Bibliothek Göttingen. Code Ms. Gauß Math. 48Cim.

I believe that this page is suitable for clearly showing us Gauss' pleasure of playing [with numbers]. There are grounds for stating that it was mainly written before the autumn of 1799, but it also includes insertions dating back to 1784 as well as later additions up to 1808. At the top of the page we find information about a walk from the gate [...] to gate [...], a table showing the times of day and therefore the time required for that walk. Under that table are some numbers, the letter B and fragments of two words. Then follow the words *Newton's Epitaph* and two lines by Pope:

Nature & nature's laws lay hidden in night. God said, Let Newton be & all was light.

So early had Gauss' admiration for Newton been manifested. [...] At the left margin there is a table compiled on 6 April 1801 which indicated the time required for a walk from [...] in Braunschweig to

[...]. We see that Gauss was a fast pedestrian who did not shuffle his feet, his marching speed was about 5.6 km/h.

In the middle of the page there is a table showing the *rounded off distance* counted in steps (1 step = 0.75 m) from Braunschweig to Helmstedt separated in eight intervals, probably estimated or reckoned by a map. (His exact *count* of those distances in steps is on another page.) [...]

Also in the middle of the page there is a table providing the mileage of the trips until the autumn of 1799 separated into walks, trips in waggons and on horseback [...], 239 miles in all. [...]

[8] So what is new, where can we see the elements of the changed understanding of Gauss? We see now a human being experiencing pleasure in playing [with numbers] but not as a superman. Only now his motives and actions became clear, but had it not diminished his greatness, or the fascination he holds for us? On the contrary. The admiration for his achievements which to a large extent depended on the atmosphere surrounding him, only strengthened since now we know and understand that he compiled his immortal contributions under circumstances which, according to his feelings, did not at all foster mental efforts. Hard work under hindering circumstances constituted a considerably greater part [in achieving success] as compared with brilliant intuition than it was admitted previously.

Indeed, the vulnerable, receptive, sensitive man had to wrestle not only with those unfavourable conditions but with himself just as well. The Gauss biography compiled by Sartorius will always remain a valuable primary source but it ought to critically used and supplemented by other sources.

This is especially true regarding the interpretation of Gauss' political views. Only weak initial signs of a new understanding have emerged. Until this day there dominates a conviction based on the report of Sartorius that Gauss, having been inspired by the demand of the Duke of Braunschweig, was (became?) a conservative and disliked any changes.

[9] A minor sensation occurred when some years ago it was discovered that two men from Gauss' immediate surroundings, whom he wholly trusted, namely his mechanic [specialist in astronomical and geodetic instruments] Moritz Meyerstein and his colleague and former student Moritz Abraham Stern, belonged to the circle of friends of Paris left radicals.

And now Gauss' statement of 20 April of the revolutionary year 1848 in a letter to Bolyai, a friend of his youth, can be seen in a new light:

The powerful political and social earthquake which extends ever wider and overturns every European custom (until now your fatherland understood in a strict sense, I mean Transilvania, is not yet affected). Nevertheless, I confidently feel that after all pleasant fruit will appear, but the transitional period will at first cause much distress and (quod tamen deus avortat [God forbid]) can last a long time. At our age it is always very doubtful whether we will live to see the Golden Age.

How to explain this statement so strikingly contrary to his other

stock remark handed down to us about revolutionary upheavals? Should not some traces of Gauss' discussions with Stern and Meyerstein be seen here? At present, we can only raise this question without answering it.

A detailed study of the life conditions of Gauss requires considerably more *knowledge about his companions*. In this connection I would especially like to recall the contribution of the members of the Gauss Gesellschaft and first of all I name the regrettably already late Martha Küssner, Horst Mischling and Dr Gresky.

I allow myself to adduce *one* example¹⁴. We know from Sartorius that Gauss had reproached Goethe for want of principles and ideals and did not appreciate too highly his *lyrical poetry*. I can show that that low appreciation had to do with a similar opinion of Goethe about Gauss. Indeed, when in 1817 Goethe had revised the comedy *Die Bestohlenen* (The Robbed) by August von Kotzebue for the stage he changed a place in the text in which Leibniz and Gauss were mentioned on a par. Kotzebue wrote: *Had you been as learned as a Leibniz or a Gauss*, but Goethe's ill humour about Gauss' silence over his theory of colours prompted him to replace Gauss by Kant: *Had you been as perfect as Leibniz and as great as Kant* (Goethe's *Jb.*, Bd. 92, 1975, pp. 195 – 219, see p. 204)¹⁵.

Future investigations of the great mass [of unpublished statements and letters] kept here in Göttingen will certainly further change his portrait. I am convinced that a new approaching understanding of Gauss will be deeper, more objective and more appropriate than the conventional hackneyed respect due to a hero. Theodor Fontane in vain warned contemporary biographers against *beautifying forever*. Nevertheless, much was irrevocably lost with Gauss' death as he himself prophesied in 1832. But even now we can safely say that our admiration for that outstanding genius, analytical power, for his purposeful persistence, his use of mathematical experiments¹⁶, his intuitive discovery of hidden connections and applications, as well as for his ensuing deepness and versatility, – that everything mentioned will remain eternally.

Notes (O. S.)

1. See Dedekind (1901/1933, p. 305) who described Gauss' lectures: *Especially clear description of the development of the main notions and main propositions of the calculus of probability*. And here is May (1972, p. 307, left column): *Teaching became less distasteful [for Gauss], perhaps because his students were better prepared and included some, such as Dedekind and Riemann, who were worthy of his efforts*.

2. I do not know anything about those dues.

3. In a letter to Gauss Legendre called himself the inventor of the method [of the principle] of least squares since he was the first to publish it. Gauss did not reply and the much older Legendre became indignant, mostly because of that silence. After that, French mathematicians dealing interested in the treatment of observations including Poisson (but not Laplace), to their own detriment, started to ignore the relevant work of Gauss. Reich (1996) stated however that at least from 1836 this attitude had changed. Legendre died in 1833. On 17 Oct. 1824, in a letter to Schumacher Gauss wrote: *With irritation and distress I [...] read that the old Legendre, an ornament to his nation and his time, was deprived of his pension*.

4. Earlier noticed by Dunnington (1955, p. 348). The commemorative medal was

issued just after Gauss' death. Its inscription read (in translation): *George V, King of Hanover, to the Prince of mathematicians*. During 2005, a century and a half after Gauss' death, the newspaper *Göttinger Tagesblatt* published 49 popular articles about Gauss and his works, then issued all those articles as a booklet called *Mein Gauss* (published by Gauss-Gesellschaft E. V., the place and year of publication apparently Göttingen, 2005 or 2006). I can only say that this booklet, if only obtainable, deserves to be scanned through. I am grateful to Professor Ulrich Krenkel for sending me a copy. Stamps commemorating Gauss were issued at least in Eastern Germany and in the united Germany. There also appeared a commemorative five-mark coin and a 10-mark banknote.

5. See [III].

6. Usually only Perthes is called the publisher of the *Theoria motus*, but Biermann [III, § 1] correctly named both of them: Perthes and Besser.

7. In a letter to Olbers of 30 July 1806 Gauss stated that his motto was *aut Caesar, aut nihil*.

8. Gauss is known to enthusiastically appraise Euler's achievements.

I am not satisfied with Biermann's conclusion about the attitudes of Gauss. *First*, Gauss had published two classical contributions, the *Disquisitiones arithmeticae* of 1801, and *Theoria motus* ... of 1809, both of them perfect or almost so, in form and essence, so he was probably quite unwilling to lessen his standard. Indicative is his explanation (1807, p. 161) of delaying the latter:

Many esteemed astronomers insistently asked me to publish the method that I had applied [for rediscovering, in 1801, the minor planet Ceres], but [various circumstances] as well as my intention to treat this matter in detail and my hope that further studies [...] will offer an opportunity to bring various parts of the method to a higher degree of perfection, generality and ease, are the causes why I am only now satisfying those friends.

It is worth noting that Gauss had certainly encountered difficulties in translating his text from German into Latin. Indeed, much later, when preparing his Latin memoir of 1823 for publication (and calling it by its finally abandoned title *New justification of the method of least squares*), Gauss (G – O, 14 Apr. 1819) remarked that *The brittle Latin language often resists natural effortless expression of thoughts*.

May (1972, p. 309, right column) inconclusively stated that Gauss *did have high standards* but published *all that was ready for publication by normal standards*. Anyway, discussing Gauss' memoir (1823), Stewart (1995, p. 222) reasonably decided: *It requires great generosity on the part of the reader to conclude that he actually proves anything* [in his §§ 12 and 13].

In § 7 and in the beginning of § 8 Biermann concludes that Gauss experienced pleasure in playing [with numbers]; elsewhere [III] he added that playing soothed him. I venture to suggest that by introducing numbers he transferred irregularity into order (his counts of primes or of people struck down by lightning [III, § 2]), and order, perfection was what he wished to see in his manuscripts. Finally, he valued harmony in the results of geodetic measurements (Gaede 1885, p. 180).

These considerations stress the *otherwise* in Biermann's statement that Gauss had *conscientiously or otherwise powerfully assisted* in portraying himself as a marble statue. Then, Biermann [III] reasonably remarked that, when collecting scientific or even useless data, Gauss attempted to order apparently random occurrences. This circumstance could have strengthened his desire for perfection.

Certainly, however, that Gauss was a *scientific despot* (Humboldt, end of § 6), a *scientific aristocrat* (Biermann [I, § 8]), a *crass egoist* (Bessel [I, § 5]). Indeed, how else can we explain his inhuman demand imposed on his sons (Note 10)? Recall also Note 7.

Second, below, in § 9, Biermann notes that Gauss eagerly wished that the revolution of 1848 will eventually bring about the Golden Age, that he was not a conservative at all (as stated by Sartorius). Here is an unjustified contradiction (perhaps issuing from Sartorius as well): *During the revolution of 1848 Gauss stood guard with the royalists* (May 1972, p. 307 left column). And (May, p. 309 left column) Gauss was *hostile or indifferent to radical ideas in mathematics as in politics*.

Concerning mathematics, I adduce a sudden comparison of Gauss with Chebyshev (Novikov 2002, p. 330):

Endowed with a brilliant analytical talent, he was a pathological conservative. He

scornfully spoke about the newfangled disciplines like the Riemannian geometry and complex analysis.

9. Klein (1926, pp. 11 – 12) noted that Gauss had sometimes suffered from morbid depression. Bashmakova et al (1978/2001, 51) quoted a comment on Gauss' *Mathematical Diary* from Klein (p. 33):

Here we see not the inaccessible, closed, cautious Gauss as he appears in his published papers. Here we see what Gauss was like when he experienced and conceived his great discoveries. He expresses his joy and pleasure in the liveliest manner, bestows laudatory epithets upon himself, and shows his mood in enthusiastic exclamations.

10. Gauss's sons reported that he discouraged them from going into science on the ground that he did not want any second-rate work associated with his name (May 1972, p. 308 right column). An inhuman demand!

11. However, Gauss highly appreciated Jacobi (and Dirichlet, to whom he had not referred either). He attributed to the former *sagacity, penetration and elegance*, see his letter to Crelle of 1828 as reported by May (1972, p. 304 right column). He was also much impressed by Dirichlet's *eminent talent*, see his letter to Encke of 8 July 1826 as reported there also. Now, Gauss *wrote and spoke for posterity* and regarded these utterances as publications (Biermann, the very beginning of § 4).

Nevertheless, *Gauss typically acknowledged the help of Weber* [in compiling an important contribution on terrestrial magnetism] *but did not include him as joint author* (May 1972, p. 305 right column).

12. Biermann had decoded some notes written by Gauss, and now, in turn, readers should decode his description. The number 64,768 is the number of days from the introduction of the Gregorian calendar to the birth of Gauss [III, § 5].

The Rosetta Stone enabled to decode ancient Egyptian hieroglyphs since its inscription also contained the text in ancient Greek (and a Demotic script as well).

13. See the very beginning of § 1.

14. Goethe was not a companion of Gauss (cf. above).

15. *Leibniz and Kant* seems more proper than *Leibniz and Gauss*. Then, Biermann did not prove that Gauss knew about the described episode. Finally, May (1972, p. 307, right column) stated that Gauss had a *rather narrow cultural outlook* and that (p. 309, left column) *did not care for Byron or Shakespeare [...], disliked Goethe and disapproved of Schuller*.

16. Biermann mentioned mathematical experiments in § 2 as well, but he probably meant empirical calculations.

Brief Information about Those Mentioned

Eckermann Johann Peter, 1792 – 1854, Secretary of Goethe and his friend

Encke Johann Franz, 1791 – 1865, astronomer

Ewald Heinrich, 1803 – 1875), orientalist and theologian, Gauss son in law

Fontane Theodor, 1819 – 1898, writer

Harding Karl Ludwig, 1765 – 1834, astronomer

Kotzebu August von, 1761 – 1819, playwright, writer

Lindenau Bernhard August von, 1780 – 1854, astronomer, lawyer, politician

Listing Johann Benedikt, 1808 – 1882, physicist

Meyerstein Moritz, constructor of optical instruments

Sartorius Waltershausen Wolfgang von, 1809 – 1876), mineralogist, geologist

Steinheil Carl August von, 1801 – 1870, physicist, inventor, astronomer

Stern Moritz Abraham, 1807 – 1894, mathematician

Wagner Rudolf, 1805 – 1864, physiologist, anthropologist

Weber Wilhelm Eduard, 1804 – 1891, physicist

Bibliography

- Bashmakova I. G., Rudakov A. N., assisted by A. N. Parshin, E. I. Slavutin** (1978, in Russian), Algebra and algebraic number theory. A chapter (translated by A. Shenitzer & H. Grant or either of them) from *Mathematics of the 19th Century*, vol. 1. Editors, A. N. Kolmogorov, A. P. Youshkevich. Basel, 2001, pp. 35 – 86.
- Biermann K.-R.** (1978), Gauss als Persönlichkeit. *Abh. Akad. Wiss. DDR*, No. 3, pp. 39 – 49.
- Dedekind R.** (1901), Gauss in seiner Vorlesung über die Methode der kleinsten Quadrate. *Ges. math. Werke*, Bd. 2. Braunschweig, 1931, pp. 293 – 306.
- Dunnington G. W.** (1955), *C. F. Gauss, Titan of Science*. New York. [Math. Assoc. America, 2004.]
- Gauss C. F.** (1807), Deutscher Entwurf der Einleitung zur *Theoria motus ... W-12*, pp. 156 – 162.
- (1823, in Latin), Theory of combination of observations least subject to error. Latin and English. Translated with Afterword by G. W. Stewart. Philadelphia.
- (1863 – 1930), *Werke*, Bde 1 – 12. Hildesheim, 1973 – 1981.
- (1975 – 1987), *Werke*, Ergänzungsreihe, Bde 1 – 5. Hildesheim.
- (1976), *Gauss' mathematisches Tagebuch*. Leipzig. Ostwald Klassiker, No. 256. Leipzig, 1985.
- Gaede** (1885), Beiträge zur Kenntnis von Gauss' praktisch-geodätischen Arbeiten. *Z. f. Vermessungswesen*, Bd. 14, pp. 113 – 137, 145 – 157, 161 – 173, 177 – 207, 225 – 245.
- Gerardi T., Editor** (1964), *Ch. L. Gerling an C. F. Gauss*. Göttingen.
- (1969), *Nachträge zum Briefwechsel zwischen C. F. Gauß und H. Ch. Schumacher*. Göttingen.
- Klein F.** (1926), *Vorlesungen über die Entwicklung der Mathematik im 19. Jahrhundert*, Tl. 1. Berlin.
- May K. O.** (1972), Gauss. *Dict. Scient. Biogr.*, vol. 5, pp. 298 – 315.
- Novikov S. P.** (2002, in Russian), The second half of the 20th century and its result: the crisis of the physical and mathematical community. *Istoriko-Matematicheskie Issledovania*, vol 7 (42), pp. 326 – 356. **S, G**, 78.
- Reich K.** (1996), Frankreich und Gauss, Gauss und Frankreich. *Berichte zur Wissenschaftsgeschichte*, Bd. 19, pp. 19 – 34.
- Stewart G. W.** (1995), See Gauss (1823).
- Worbs E.** (1955), *C. F. Gauss. Ein Lebensbild*. Leipzig.

III

An inborn player in the scientific work of C. F. Gauss

Lusus ingenii im Schaffen von C. F. Gauss.
Mitt. math. Ges. Hamburg, Bd. 12, No. 2, 1991, pp. 329 – 346

[1] *A spectacular whim of fate*, as Gauss had remarked on occasion, is that the observatories in Göttingen and Altona are situated *on the same meridian to less than the width of a house* [II, Intro.]. And a similar whim, as I may say, is that I found out by chance that Gauss was a member of the Mathematical Society of Hamburg [1, p. 8] at the same hour as I received the honourable invitation to report on a subject of my research at the tercentenary of this Society rich in tradition.

For more than 30 years I have been studying the work of Gauss so that that chance coincidence led me to write about the Prince of mathematicians. Quite apart from the fact that the subject *Gauss* can always count on attracting the attention of mathematicians, be they more or less historically minded, the Prince had maintained many-sided relations with Hamburg and Altona.

First of all I should perhaps mention that Gauss' main astronomical contribution, the *Theoria motus*, was published by Perthes & Besser in Hamburg. It was in Hamburg that Gauss had come to his first thoughts about inventing the heliotrope for reflecting sunlight and thus for serving as a sighting target for geodetic measurements. In October 1818, in Lüneberg, Gauss noticed that *the [western – K.-R. B.] window of the uppermost gallery of the Michaelis tower illuminated at that moment by the Sun was seen as a shining ray of light* [2/4.2, p. 47].

In 1821 Gauss had seriously thought about becoming director of the new observatory in Hamburg [2/4.2, p. 81]. Nothing came out of it actually for the same reason that his repeated intention to be invited to Berlin did not realize: those responsible hesitated and economized. Edmund Landau stated on occasion that the mathematical centre of the German language area was situated *in the triangle Göttingen – Berlin – Hamburg* [3, p. 202]. Allowing a free rein of imagination, we may speculate how would that centre shifted had Gauss really moved to Hamburg.

We should also mention Gauss' visits to Hamburg and Altona, and first of all his correspondents there certainly beginning with his closest friend, the Altona astronomer Heinrich Christian Schumacher, the founder and first editor of the still existing *Astronomische Nachrichten*. He had also played a certain role in the history of mathematics as well. We may recall that the dramatic contest between Abel and Jacobi in the construction of the theory of elliptic functions was partly held on the pages of that periodical.

It is also possible to mention Repsold, the highly esteemed by Gauss manufacturer of astronomical instruments. He was also the head of the city fire brigade and lost his life in the great fire of 1830. I can, but will not also name half a dozen other correspondents from Hamburg and Altona, but the mentioned above is sufficient proof for

having good grounds to report about Gauss here and today. From the almost inexhaustible for a historian range of topics relating to Gauss I have chosen a problem area which can most of all excite listeners. Namely, I will deal with two Gauss' inclinations both of which seem playful: a predilection for recording numerical outcomes and, until age 40 or thereabouts, a tendency to encode his results.

[2] Gauss – a playing man? This will astonish those whose image of Gauss was formed from the picture systematically drawn by his trusted friends and companions in Göttingen and consciously, as I have shown [4, p. 44], assisted by himself. Such inclinations do not at all fit that picture of a *bronze block* [5, p. 45]¹.

Incidentally, at age 25 Gauss himself admitted playing with fiction when in 1802 in a letter to Franz von Zach he stated about the so-called Titius – Bode law on the mean distances of the planets from the Sun [6, p. 504 (Gauss); p. 444 (Humboldt (1850))]: *It should not be disapproved at all when such approximate coincidences are searched for in nature. Greatest men of each time have indulged in such approximate coincidences.*

And even 45 years later he [2/5.5, p. 394] wrote to his intimate friend Schumacher:

In general, I am lenient with imagined games. [...] No, I do not deny that I sometimes amuse myself in a similar way but I will never publish anything of that kind. To such amusements belong for example my thoughts about the inhabitants of celestial bodies.

However, I will dwell not on such imagined games concerning astronomical matters, but on his records and encoding as a means of unwinding and relaxation. Four categories ought to be discerned in his numerical tables and records. At first I should mention the results of observation which Gauss applied *for inductively discovering arithmetical relations* [7, p. 5), for example the table of the frequencies of primes, of *cyclotechnie*² and for decimalizing fractions [Gauss, W-2, pp. 435 – 443, 477 – 496 and 411 – 434].

A riddle is contained here: how could have the young Gauss revealed *concealed connections without some theoretical viewpoints* [7, p. 66], or [8, p. 37] *fish out from time to time number-theoretic theorems from the great pond of his tables?*

For Gauss, compilation and effective arrangement of his auxiliary tables for rapidly checking calculations [8, p. 44] was a point of taste and aesthetic pleasure. In a lecture on the method of least squares he stated half in jest that there is *certain poetry* in compiling tables of logarithms [9, p. 444], and even prolonged adjustments of geodetic measurements provided him satisfaction [2/1, p. 412].

After tables of wide number-theoretic interest we should mention Gauss' records of numbers intended to open up new applications for mathematics³ [10, p. 89]. Even in 1802, being 25 years old, Gauss expressed his hope for editing censuses, of data on births and deaths in Braunschweig both for his own *pleasure* and for becoming *useful* [2/4.1, p. 106]. Later he compiled tables which should have served for discovering regularities in the mortality of infants and people of extreme old age [10, p. 89]. In a letter to his friend Humboldt of 15 April 1846 [11, p. 95] he wrote:

Had I been a Rotschild, I would have donated a million with the interest being yearly distributed among 400 oldest inhabitants of a large country under the condition that their age and life be most perfectly studied.

In the same letter Gauss also stated that for him exact and detailed statistical data on the mortality of babies would have been *something just as (or much more) interesting as the determination of a new planetary orbit.*

And when for many years Gauss had recorded the number of aces dealt out to gamblers in each set of whist in which he himself had participated [9, p. 444], it was his intention of checking the coincidence of frequency and probability.

His record of storms [10, p. 89] was compiled owing to his intention of discovering regularities in seeming disorder. A record of the monthly receipts of the Hanover railroads (Ibidem) and the daily reading of home and foreign newspapers for registering the fluctuations of the prices of securities [9, p. 444] was founded on his aspiration to subject conjunctures and crises to calculus. We know how successful Gauss was in his studies of booms and slumps from the fact that, initially indigent and always frugally paid (to compare: Humboldt earned about six times more), he made something like 500,000 marks [12, p. 237]. By our present yardstick, and taking into account the purchasing power at those times, at the moment of death he thus became a millionaire many times over. In Göttingen, his skill in increasing his fortune was *almost proverbial* [9, p. 444].

However, Gauss was interested not only in recording suchlike data; over and above that he attempted to gain other exact figures [2/5.5, p. 325], for example about the number of people struck down by lightning and the frequency of the lightning bolts per area unit [11, p. 96].

Tables of the third category show his efforts *to base everything on numbers* [10, p. 89]. These are lists borrowed from the literature, and I mention as a typical example a list of 78 peaks and places or regions [13, p. 73] from Chimborazo [in Ecuador] to Montblanc, from Brocken to Oderbruch in Harz.

Finally, the fourth category is comprised of such tables which were compiled as a jocular amusement, and here is an example, typical in my opinion [14, sheet 8^v]. After the last page of his famous *Mathematical Diary* (Gauss 1985) in which he had recorded his findings during 1796 – 1814 *there are some sheets with both mathematical and non-mathematical statements* (W-10/1, p. 572; [15, p. 25]). [...] ⁴.

[3] Only a few words about the distances measured in steps and found on a page full of jocular elements. It is reported that Gauss, in later years as well, recorded *the distances in steps from the observatory [in Göttingen] to those places which he had visited more often* [10, p. 89]. On 31 Dec. 1837 he wrote to Schumacher [2/5.3, p. 190] that during those counts he was able to occupy himself otherwise. He read the indication of the French astronomer Lalande that an astronomer engaged in practical astronomy ought to be certain of his counts of seconds to the extent of being able to walk [a few

steps], write something down and even speak without interrupting his count or being mistaken⁵. This statement prompted Gauss to remark in the same letter:

I can do much more, I can think coherently about quite other matters, or count something quite independent from the first count or read a book or a letter. [...] However, I do not dare talking, or talking more than a few words without getting out of the count.

Table 2 also taken from an appendix [14, sheet 14^f] to the *Mathematical Diary*⁶ lists the stages [of a walk] with the relevant minutes and numbers of steps. Thus, we can imagine how Gauss, apparently in October 1798, walked from Braunschweig to Helmstedt. He came to Bornum in 180 minutes, to [...] and to Helmstedt in 370 minutes having counted 45,053 steps and at the same time thinking about, for example, his proof of the theorem that each algebraic rational whole function (?) of one variable can be expanded into real factors of the first or the second degree, – the proof [of the main theorem of algebra] that he offered a bit later in the dissertation defended in Helmstedt [16]. His speed amounted to ca. 5.7 km/h; recalling that he covered 35 km, we conclude that this should be called a sporting achievement of a 21-year-old man.

The page with the mentioned numerical results also contains various tests of the pen and a copy of a French love poem (probably written by Jean-Baptiste Rousseau) and of its German translation by Friedrich Wilhelm Gotter. The poem ends thus: *When you open your lips beats my entire heart, touching your hand jerks me to the sky.*

So Gauss had not lived only in the world of numbers at all. Further proofs of this statement are found not only in his letters [2/2, pp. 16, 61 – 62], but also in his notes. For example, in a field record book of his Braunschweig triangulation of 1803 he had repeatedly written down the name of his future first wife, Johanna Osthoff [17, pp. 15, 17], with whom he had just been acquainted, with whom he fell in love a year later, who died only five years after that, deeply mourned and never forgotten by him.

[4] Here is another appearance of an inborn player. Among the supplements to the *Math. Diary* there is a table [14, sheet 29^f] which I [18, pp. 8 – 14) have interpreted as an indication of the number of thousands in which Gauss had counted the number of primes on certain days. The compilation of this table (see Fig. 3) began on 15 Dec. 1791, when he was even younger than fifteen, earlier than Gauss (W-2.2, p. 444) had recalled almost 60 years later and it ended on 28 Nov. 1797 after Gauss had studied 56 thousands.

There are no *en clair* indications and the dates in the first column are provided not in the usual way but as four-digit numbers denoting the number of days beginning with Gauss' birth. Hints are actually offered by coincidences on the second line from above and on the sixth line from below: 97.4.15 C (chiliaden [thousands – O. S.]) A (Abzahlungen, counts) 7291, and 8113; 99 VII 16 D (Doctor).

Calculation shows that on 15 April 1797, the day up to which Gauss had studied 20 thousands, 7291 days, and on 16.7.1799, the day on which he had defended his dissertation, 8113 days have passed since his birth. The second column shows the increase in the number of the

studied thousands and the corresponding day, in the third column are those numbers since the beginning of the counts.

The fourth column contains much mysterious, for example crosses of various kinds, other signs and words. The fifth column consists of weekdays denoted by numbers, the two next columns show the day of the month and of the days of the count and therefore offer an additional means of checking. This column provides a special amusing play insofar as the weekdays are shown not in the then usual form, that is, not denoting Sunday by 1, Monday by 2 etc., but otherwise: Tuesday was 0, Wednesday was 1 etc. [...] We can only speculate why had Gauss denoted Tuesday by a zero, although possibly because he was born on a Wednesday.

[As mentioned above], Gauss had concluded his table on 28 Nov. 1797, but later he *very often spent a free quarter of an hour for studying a thousand here or there* (W-2.2, p. 445). Or, more properly: studied them in an unintended way. At first, as I said, he inserted the day of the defence of his dissertation, then another date (3 April 1801) which, according to some information, could have referred to his paper [19, pp. 136 – 140] in which he (p. 140) had derived the *condition for the existence of a limit of a countable set as a quite special case by issuing from its invariably existing upper and lower boundaries*.

Four more dates are given and an additional calculation is provided. It mainly corresponded to his geodetic measurements of 1824. We do not know why Gauss had considered his calculation important. Among other abbreviations there is the letter Z which possibly denoted Zeven, Gauss' temporary accommodation, highly valued by him in contrast to other quarters in which he lived during his triangulation measurements since 27 June 1824.

[5] Until old age Gauss had kept to his usual peculiar notation of dates by the number of days since his birth. For example, he had thus calculated Eisenstein's age at death [20, p. 7]. From Sartorius [...] we know that Gauss had *compiled a list of the duration of lives measured in days mostly of eminent people, namely, of his friends* [10, p. 89]. As far as I know, that list is not yet published, but I [21] published the *count of days* written on a blank page of his own copy of a logarithmic table of 1811 (Fig. 4). He calculated the number of days of Newton's life to find out on which date Alexander von Humboldt will arrive at Newton's age at death, – on 9 Dec. 1853. Next to his own day of birth (30 April 1777), on which 64,768 days have passed since the introduction of the Gregorian calendar [in 1582 – O. S.]⁷ Gauss also found place in his table for indicating the day of his dissertation's golden jubilee.

Basing himself on this method of dating, Gauss [11, pp. 113 – 114] had stated in his letter of congratulation:

We, Germans, celebrate with pleasure, perhaps more than any other nation, certain days which have some temporal connection with our dear people or events such as birthdays, jubilees, a. o.

Even now this rings very topical. Gauss continued:

Representatives of the quantitative science, in whose eyes indefiniteness and arbitrariness are always considered repulsive as

opposed to clarity and stability, find a small deficiency in that the ground for establishing for celebration exactly this day rather than another one more or less depends on arbitrariness [...] and, in the final analysis, on the circumstance that we have exactly five fingers to each hand.

Humboldt's joy over the *astonishing* congratulation on the occasion of his *dreadful 30,766 days of life* was restricted (who would be pleased to be reminded of his old age?) but he discerned *something peculiar to the great man* [21, p. 165].

[6] I doubt that avoidance of arbitrariness was the only decisive argument for the fifteen-year-old Gauss when he began to provide dates of his life measured in days from his birth. I rather believe that it already was his inclination to encode, his tendency to erase each trace (to follow Kronecker [22, p. 42]) and, as Philipp Maennchen [23, p. 105] had put it, *to insert complications even in jokes.*

Along with the search for the primes' law of distribution the young Gauss had been mostly fascinated by *playing with the arithmetic-geometrical mean* [24, p. 45]. I will only briefly dwell on his relevant notes as far as they were *intentionally compiled in a puzzling form* [19, p. 12].

From 1796 until 1816 Gauss had been without explanation using artificial words such as GEGAN, WAEGEGAN, GALEN and groups of letters, for example WAE AZ ACLN L in his *Math. Diary* and notebooks. I [25; 26; 27] have attempted to find out plausibly that all those letters relate in various ways to his great discovery of the connection between the lemniscate, arithmetic-geometrical mean and power series as well as to the resulting elements of the general theory of elliptical and modular functions. I will not go into details since they are documented in my publications (Ibidem).

Being based on circumstantial attempts at interpretation, they are inevitably hypothetical and therefore questionable. It is thus understandable that other explanations are offered [28]. They coincide with my interpretation insofar as they also issue from a connection between the artificial words and the arithmetic-geometrical mean, but the essence of their statements is very general and, most important, the freely existing inner interrelation of all the decoded words is lost.

My assumption that by his encoding Gauss had attempted to prevent outsiders from gaining an insight into his mental workshop has also been criticized. It was accepted that,

when working under great stress, or being enormously joyful over discoveries, Gauss had no time or inclination for formulations in detail and in such cases he often used abbreviations [28, p. 18].

However, exactly the attention paid by Gauss to write down his keywords or encoded combinations of letters in *adorned capital letters* [29, p. 24] to a certain extent indicated the possibility of leisure. The danger that some outsider in Braunschweig was able to gain an insight into his notes certainly did not exist and in Göttingen that danger hardly existed. But it was exactly *the acting as though* that appealed to him. I therefore consider Gauss' inclination to encode not as a corollary of his attempt to economize time, but as the act of an *inborn player*. For someone as extremely skilful at, and experienced in

calculations as Gauss was, this attitude led to an increase of the required time; instead of the usual dating he had to find out the number of days passed from his or someone else's day of birth⁸.

In this connection it should be mentioned that in 1812 Gauss had deviated from his principle of publishing only quite ripe materials [2/5.2, p. 94; 30, p. 40]: he made known an encoded [31] conclusion in which he was not quite sure. It was the only occasion on which he had revealed *coram publico* [to the public – O. S.] his inclination to encode. His cryptogram should have meant that *the main motions of Jupiter and Pallas are in a rational ratio of 7:18* [2/1, p. 170]. My published modest attempt at decoding [32] differs from other endeavours [32, p. 156] in that I understand the encoded message not only as stated above, but as also including the date of the discovery, 3 April 1812.

I hope that my explanation has thrown light on two points: Gauss liked to deal with numbers even without setting objectives since it entertained and soothed him; and, until reaching maturity he had a weakness for encoding. Both inclinations expressed his strive for playing.

I also wish to indicate that even today it makes sense to study the unpublished notes of that probably unique genius. Such work can be essentially eased by the publication of the catalogue of his manuscripts kept at the Staats- und Unibibliothek Göttingen. Its author, my friend Theo Gerardy (1908 – 1986) had not completed it. I would like to drop a hint for simplifying this work. We may assume that, while attempting to economize on expensive writing paper, on blank spaces [for example, in published tables] Gauss had written the results of his collateral calculations, as they are thought to be, and then inserted four- or five-digit numbers with abbreviations which had no connection with the initial aim of the record. Actually, they denote dates of his life and contributions or of the lives of others. Quite generally and unchangeably valid is still the statement [33, p. 73]

*It is really probable that the scope of important ideas is not yet understood and will only become fruitful in the future*⁹.

Nevertheless, as Gauss prophetically foresaw in 1832, much had been irrevocably lost with his death [30, p. 41]¹⁰.

Notes (O. S.)

1. See [II].
2. Cyclotechnie is connected with the expansion of numbers into products of primes, see explanation in the source mentioned.
3. Application of mathematics (more precisely, of the theory of probability) to demography, see below, was not new at all. I (Sheynin 1979, pp. 81 – 63) have described Gauss' study of the laws of mortality.
4. I have omitted more than a page of the author's text also contained elsewhere [II, § 7].
5. Chronographs were still unknown and observers had to use the method called *eye – ear*; I myself used it while being a student of the Moscow geodetic Institute. The observer memorizes the indication of his chronometer and simultaneously counts the seconds according to its ticking, then observes and registers the passage of a star across the crosshairs of the ocular of his instrument.
6. I have omitted both the tables and the reproduction of pages from Gauss' *Mathematical Diary*.
7. Catholic Europe officially passed on to the Gregorian calendar in 1582;

actually, however, European countries introduced it later (and not at all simultaneously).

8. Note however that the lost time was more important for an able calculator. Maennchen (1918b) stated that Gauss had often made mistakes in his calculations since he did not check himself (apparently in less important cases).

9. Possibly Yang Qing Zhi et al (1997) is here useful. Now, however, this source is not easy to get hold of.

10. Gauss foresaw it in 1822 [I, Note 6].

Brief Information about Those Mentioned

Eckermann Johann Peter, 1792 – 1854, Goethe's secretary and friend

Eisenstein Ferdinand Gotthold Max, 1823 – 1852, mathematician

Götter Friedrich Wilhelm, 1746 – 1797, poet

Landau Edmund Georg Hermann, 1877 – 1938, mathematician

Sartorius Waltershausen Wolfgang von, 1809 – 1876, mineralogist, geologist

Zach Franz Xaver von, 1754 – 1832), astronomer

Bibliography

Note: Materialen (1911 – 1920) is not definite enough. Their issues have been published either as books or as papers in periodicals. And I have found *Materialen* which appeared after 1920.

[1] **Michling H.** (1979), Aus der Bücherei des Gymnasiasten J. F. C. Gauß. *Mitt. Gauß-Ges.*, No. 16, pp. 5 – 16. Johann was a name which Gauss never used.

[2] **Gauß C. F.** (1975 – 1987), *W/Erg-1 – 5.*

[2/1] *Briefwechsel mit Bessel*, Bd. 1,

[2/2] *Briefwechsel mit Bolyai*, Bd. 2,

[2/3] *Briefwechsel mit Gerling*, Bd. 3,

[2/4] *Briefwechsel mit Olbers*, Bd. 4/1, 4/2,

[2/5] *Briefwechsel mit Schumacher*, Bd. 5/1, 5/2, 5/3.

[3] **Biermann K. R.** (1988), *Die Mathematik und ihre Dozenten an der Berliner Universität 1810 – 1933*. Berlin.

[4] **Biermann K. R.** (1978), Gauß als Persönlichkeit etc. *Abh. Akad. Wiss. DDR*, No. 3N, pp. 39 – 49.

[5] **Küssner Martha** (1977), Gauß' Umzug von Braunschweig nach Göttingen etc. *Mitt. Gauß-Ges.*, No. 14, pp. 30 – 47.

[6] **Gauss C. F.** (1802), Brief an Fr. von Zach 16 Okt. 1802. *Monatl. Corr.*, Bd. 6, p. 504; Humboldt A. (1850, p. 444).

[7] **Bachmann P.** (1922), Über Gauss' zahlentheoretische Arbeiten. *W-10/2*, No. 1.

[8] **Maennchen Ph.** (1918a), Die Wechselwirkung zwischen Zahlenrechnen und Zahlentheorie bei C. F. Gauss. In *Materialien* (1911 – 1920), see issue of 1918, No. 6).

[9] **Cantor M.** (1878), C. F. Gauss. *Allg. Deutsche Biogr.*, Bd. 8, pp. 430 – 445.

[10] **Sartorius von Waltershausen W.** (1856), *Gauss zum Gedächtnis*. Wiesbaden, 1965.

[11] **Biermann K. R., Editor** (1977a), *Briefwechsel zwischen Humboldt und Gauss*. Berlin.

[12] **Dunnington G. W.** (1955), *C. F. Gauss, Titan of Science*. New York. [Math. Assoc. America, 2004.]

[13] **Reich Karin** (1977), *C. F. Gauss 1777/1977*. München.

[14] Staats- u. Uni. Bibl. Göttingen, Gauss Math. 48 Cim.

[15] **Biermann K.-R.** (1986), Wissenschaftliche Beziehungen von C. F. Gauss. *Sitz. Ber. Öster. Akad. Wiss.*, Abt. II, 195, No. 1 – 3, pp. 25 – 40.

[16] **Gauss C. F.** (1799), *Demonstratio nova theorematum omnium functionum algebraicarum racionales integram unius variabilis in factores reales primi vel secundi gradus resolvi posse*. *W-3*, pp. 1 – 30.

- [17] **Gerardy T.** (1977), Die Anfänge von Gauss' geodätischer Tätigkeit. *Z. Vermessungswesen*, Bd. 102, No. 1, pp. 1 – 20.
- [18] **Biermann K. R.** (1977b), Aus unveröffentlichten Aufzeichnungen des jungen Gauss. *Wiss. Z. Techn. Hochschule Ilmenau*, Bd. 23, No. 4, pp. 7 – 24.
- [19] **Schlesinger L.** (1912b), Über Gauss' Arbeiten zur Funktionentheorie. In *Materialien* (1911 – 120), issue of 1912, No. 3.
- [20] **Galle A.** (1917), Gauss als Zahlenrechner. In *Materialien* (1911 – 1920), issue of 1919, No. 4.
- [21] **Biermann K. R.** (1974), Humboldt zu Newton in Beziehung gesetzt durch Gauss. *Mitt. math. Ges. DDR*, No. 1 – 2, pp. 162 – 167. Staats- u. Uni. Bibl. Göttingen 8° Cod. Ms. philos. 34, p. 4.
- [22] **Kronecker L.** (1901), *Vorlesungen über Mathematik*, Tl. 2/1. Leipzig.
- [23] **Maennchen Ph.** (1934), Zur Lösung eines rätselhaften Gaußschen Annagramms. *Unterrichtsbl. Math. u. Naturwiss.*, Bd. 40, pp. 104 – 106.
- [24] = [8]
- [25] **Biermann K. R.** (1963), Zwei ungeklärte Schlüsselwörter von C. F. Gauss. *Monatsber. Deutsch. Akad. Wiss.*, Bd. 5, pp. 241 – 244.
- [26] **Biermann K. R.** (1969), Versuch der Deutung einer Gaußschen Chiffre. *Ibidem*, Bd. 11, pp. 526 – 530.
- [27] **Biermann K. R.** (1976), Schlüsselwörter bei Gauss. *Arch. Intern. Hist. Sci.*, t. 26, No. 99, pp. 264 – 267.
- [28] **Schumann** (1976), Vicimus GEGAN, Interpretationsvarianten zu einer Tagebuchnotiz von C. F. Gauss. In *Intern. Z. Geschichte Ethik, Naturwiss., Techn. Med.* (NTM), Bd. 13, p. 17 – 20.
- [29] **Schlesinger L.** (1912a), C. F. Gauss. Fragmente zur Theorie des arithmetisch-geometrischen Mittels etc. In *Materialien* (1911 – 1920), issue of 1912, No. 2.
- [30] **Schering E.** (1887), *Gauss und die Erforschung des Erdmagnetismus*. Göttingen.
- [31] **Gauss C. F.** (1812), Über Störungen der Pallas. *Gött. gel. Anz.*, pp. 657 – 660.
- [32] **Biermann K. R.** (1971), Zum Gaußschen Kryptogramm von 1812. *Monatsber. Deutsch. Akad. Wiss.*, Bd. 13, pp. 152 – 157.
- [33] **Rieger G. J.** (1957), Die Zahlentheorie bei C. F. Gauss. In *C. F. Gauss. Gedenkband zum 100. Todestag*. Leipzig, pp. 37 – 77.
- [34] **Biermann K. R.** (1986), Wissenschaftliche Beziehungen von Gauss. *Sitzungsber. Österr. Akad. Wiss.*, Abt. II, Bd. 195, No. 1 – 3, pp. 25 – 40.
- Gauss C. F.** (1985), *Gauss' mathematisches Tagebuch*. Leipzig. *Ostwald Klassiker*, No. 256. Leipzig.
- Humboldt A. von** (1850), *Kosmos*, Bd. 3. Stuttgart.
- Maennchen Ph.** (1918b), Gauss als Zahlenrechner. *W-10/2*, No. 6.
- Materialien** (1911 – 1920), *Materialien für eine wiss. Biographie von Gauss*. Eight issues prepared by F. Klein, M. Brendel, and L. Schlesinger.
- Sheynin O.** (1972), C. F. Gauss and the theory of errors. *Arch. Hist. Ex. Sci.*, vol. 20, pp. 21 – 72.
- Yang Qing Zhi et al** (1997, in Chinese, Engl. summary), Analysis of the reasons why many of Gauss' mathematical ideas were never published. *Math. Rev.* 1998j.

IV

C. F. Gauss

A Sketch of the Introduction to the German text of the *Theoria motus* (an Excerpt)

Deutscher Entwurf der Einleitung zur *Theoria motus* (1807).
Werke, Bd. 12, 1929, pp. 156 – 162

During a few weeks after the discovery of Ceres its orbit became known only along an arc covering 3° of its geocentric motion, and after a year Ceres had to be searched for in a quite another part of the sky.

I first applied my method in October 1801, and, by using the result derived from it, Ceres was found during the first cloudless night exactly there, where it was looked for [on 7 Dec. 1801, by von Zach – Editor]. In a short while, the second, the third, and the fourth new planet provided a further possibility of checking the general applicability of my method.

Soon after the rediscovery of Ceres many eminent astronomers began to ask me insistently to publish my method. However, various hindrances, my wish to expound thoroughly this subject, and, finally, my hope that a further occupation with these works will bring the various parts of my method to a higher degree of perfection, generality and handiness, only now allowed me to satisfy the desire of those friends of mine. I flatter myself with hope that that delay will not cause their discontent.

During the passed time I had very much repeatedly changed my initial method, added a great deal and in many of its parts followed quite another ways. Little in common is left between my initial method of calculating the planetary orbits and that which I applied in this work. I certainly had not intended to offer a complete account of my investigations, but neither had I thought about completely excluding many of my previous methods, the less so since they concerned the solution of exceptionally interesting problems. On the contrary, along with the really easiest and most useful methods of solving the intended main problem, I collected everything, which, during considerably long calculations, I found remarkable and practically tested about the motion of the heavenly bodies. Nevertheless, I invariably describe my own (*eigentümliche*) investigations in more detail and touch on the known in so far as it is necessary for the completeness of the whole.

This work therefore naturally breaks down into two sections. The first one is devoted to the study of all the most interesting and most useful relations between the various magnitudes which describe the motion of the heavenly bodies around the sun according to the Keplerian laws. In addition, this study prompts many peculiar methods for deriving geocentric phenomena from the elements. Those phenomena result from the complicated (*künstlich verwickelten*) combination of the elements and it is therefore necessary first of all to get confidently acquainted with all the separate tangles of that web,

then dare to hope once more to take successfully apart the individual threads and unravel the whole into its initial separate parts.

In the second section, it will be so much easier to solve the inverse problem, namely, to derive the elements from the phenomena, since the greatest part of the necessary individual operations is already known from the first section, and the work mostly reduces to collecting, ordering and combining them in a common whole.

I have accompanied most problems by examples choosing them when possible from really occurred cases. Hopefully, they will prove the practical usefulness of the solutions and illustrate them. Because of the increased handiness, less proficient readers will also be able to acquaint themselves with the whole, and the number of the adherents of these calculations, which comprise one of the most important and most splendid branch of theoretical astronomy, will increase.

Editor's Remark

In the autumn of 1806 Gauss had begun working out his *Theoria motus*, and, approximately in April 1807 its German text was ready (see his letters to Olbers of 29 Sept. 1806 and 28 April 1807). He still had no publisher, and Olbers turned to the Hamburg bookseller Perthes. At first, Perthes declined, then stated that he was prepared to publish that work *in Latin* (see the letters of Olbers to Gauss of 21/22 April and 6/7 May 1807). Gauss agreed and began the translation at once (his letter to Olbers of 26 May 1807). In November 1807 the printing began, but the going was slow, and the work only ended in June 1809 (letter to Olbers of 27 July 1809). Only the sketch published here is left from the initial manuscript written in German.

Brendel

O. Sheynin

A Little Known Side of Gauss

1. The Marble Statue

Biermann [II] traced the change of our image of Gauss: his marble statue gradually became a human being with his *contradictions, doubts and attempts, not free from his moods, sufferings and struggles*. The sculptors of that cold statue belonged to the inner circle of Gauss' surroundings during the last two decades of his life, but the main sculptor was Sartorius von Waltershausen. Biermann also stated that Gauss had *conscientiously or otherwise powerfully assisted* those attempts. I somewhat differ.

First, Gauss would have been unable to conceal the encountered difficulties and troubles or his helplessness in everyday life. Second, even when restricting Biermann's conclusion to the realm of science, there is much to say about it. In 1801, Gauss published the *Disquisitione arithmeticae* which immediately made him one of the first (if not the best) mathematician of the whole world and in 1809 appeared his *Theoria motus*, a masterpiece of astronomy.

Understandably, Gauss did not wish to lower the scientific level of his work and indeed, on 30 July 1806 (even before the *Theoria motus* was published) he made known his motto in a letter to Olbers: he intended to be *either Caesar or a nonentity*. Then, Gauss is known to have been collecting information, non-scientific as well as scientific, with a view of arranging random or only seemingly random events and discovering some order¹. Biermann [III] reasonably noted that this habit could have well strengthened his desire for perfection. I conclude that Gauss had indeed unconscientiously and *unavoidably assisted* in sculpting that marble statue.

2. Unpleasant Features

Humboldt called Gauss a *scientific despot* (Biermann [II], without an exact reference) and Bessel (Biermann [I] considered him an *insensitive egoist*. Indeed, in 1833 Gauss published an essential contribution on terrestrial magnetism, *typically acknowledged the help of Weber but did not include him as a joint author* (May 1972, p. 305, right column) and his sons by his second marriage stated (Ibidem, p. 308, right column) that he *had discouraged them from going into science* [since] *he did not want any second-rate work associated with his name*. May (p. 307, right column) also indicated *personal ambition* (along with *intellectual isolation*) and *deep conservatism*. Indeed (p. 309, left column) Gauss was *hostile or indifferent to radical ideas in mathematics*, which, however, was somewhat far-fetched since Gauss is known to have studied the *anti-Euclidian geometry* (although May stated that Gauss had disliked and suppressed it). And here is a sudden comparison of Gauss and Chebyshev: the latter was a *pathological conservator* (Novikov 2002, p. 330)².

3. References to Other authors

Biermann [I] described Gauss' reluctance to refer to other authors. In particular, he (certainly being preceded by other commentators) quoted C. G. J. Jacobi who had remarked that *for over twenty years Gauss had never quoted either me or D [Dirichlet]*. At the same time, in his correspondence Gauss, however, put a high value on both these scholars (May 1972, p. 304, right column).

Biermann [I] also quotes Gauss: he, Gauss, refers to other authors only after convincing himself of their merit, but he has neither time nor inclination for literary studies.

However, Gauss had a few times mistakenly referred to others which could have strengthened his resolve as stated above. Thus, in 1770, Boscovich had offered a certain method of treating observations and Gauss (1809, § 186) mentioned him and mistakenly stated that Laplace had modified that method. There also, in § 177, Gauss attributed to Laplace rather than to Euler the computation of the integral of the exponential function of a negative square. Later, as Börsch and Simon, the Editors of Gauss (1887, p. 207), noted, he revealed his mistake but did not correct it since Euler had not presented that integral in its final form and, which was more important, a correction was undesirable since the material was in print.

4. Imperfect Contributions

The Note of 1810. It appeared in a six-volume encyclopaedia on the history of literature (1805 – 1813) which, however, included items on natural science and mathematics. Its Editor was J. C. Eichhorn, a professor at Göttingen, who asked Gauss to describe mathematics and astronomy in the 18th century Germany. Biermann (1983), who reprinted the note, reasonably remarked that Gauss had to overcome his dislike of writing popular accounts and to satisfy that request.

Gauss almost failed. He insufficiently described the merits of Lambert and Daniel Bernoulli and called Süßmilch a mathematician. Germany (Biermann, p. 427) was then thought to comprise the region of the German language³, but Lambert called himself a Swiss (Wolf 1860, beginning of essay). I do not know whether Jakob and/or Johann Bernoulli considered themselves German or Swiss, but Euler (whom Gauss highly praised) was partly a Russian scholar. Herschel (see below), whom Gauss also called a German scientist, was after all an English scholar. Moreover, why then Gauss had not mentioned German scholars working in Russia (e. g., Goldbach)?

Gauss (Biermann 1983, p. 426) indicated that during the 18th century four German scholars (he named only three, Herschel, Olbers and Harding) had discovered five planets whereas Herschel had also discovered six satellites of Uranus. The five planets were Uranus and four minor planets (not thus called in those times and discovered in the very beginning of the 19th century). However, Herschel had indeed discovered Uranus, but thought that this heavenly body was a comet. Even now only five of its satellites are known of which Herschel had discovered only two.

The Memoir of 1823. Some places there are still incomprehensible. Here is Stewart (1995, p. 222) about its §§ 12 and 13:

It requires great generosity on the part of the reader to conclude that he [Gauss] actually proved anything.

A special point here is that the principle of least squares can be derived without any intermediate considerations (as in §§ 12 and 13). In § 6 Gauss introduced the density (though not the term) calling it the measure of precision for continuous densities. At the end of the memoir he proved, which was not difficult, that the sample variance is proportional to the sum of the squares of the residual free terms of the adjusted system of equations. Gauss thus arrived at the principle of least squares but did not even hint at this possibility. Why? Such was his well known habit, and I need not go here into details. See Sheynin (2012).

The Memoir of 1828. On p. 152 Gauss indicated that he was determining for the second time the latitudinal difference between the observatories in Göttingen and Altona but he did not say anything about its first determination. In several tables of the results of observations 16 stars remained unnamed without any explanation. In two cases (pp. 172 and 189) Gauss calculated the probable error of some results only tacitly assuming the appropriate normal distributions. On p. 161 Gauss called the arithmetic mean the most probable estimator (which it indeed is, but only for normal distributions) although in 1823 he turned instead to most reliable estimators. Finally, Gauss (p. 177) not quite properly equated residual free terms of an initial system of equations with errors. The same, however, can be said about Legendre and Laplace.

5. The Problem of Priority

To Gauss (May 1972, p. 309)

Priority meant being first to discover, not first to publish; and he was satisfied to establish his dates by private records, correspondence, cryptic remarks in publications.

The most important case here was his discovery of the principle (and calling it *method*) of least squares. Gauss indicated that Legendre had priority of publication but claimed it for himself, since he had applied it from 1794 or 1795.

Legendre had protested whereas Gauss, about 25 years younger, did not answer his letter. As a result, for a long time French mathematicians including Poisson but not Laplace did not mention the appropriate works of Gauss. All that could have been different if only Gauss had answered Legendre, or, even better, if Legendre, instead of writing to Gauss, would have remarked at a later occasion, that everyone will agree with him rather than with Gauss. And here is the final stroke (letter of Gauss to Schumacher of 17 Oct. 1824):

With irritation and distress I have read that the pension of the old Legendre, an ornament to his nation and age, was cut off.

Notes

1. On the inductive discovery of arithmetic regularities see Bachmann (1922).
2. And his talented student Liapunov (1895/1946, pp. 19 – 20) called Riemann's ideas *extremely abstract*, his investigations *pseudo-geometric* and sometimes, again, too abstract and having nothing in common with Lobachevsky's *deep geometric studies*. Nevertheless, he indirectly recalled Klein, who, in 1871, presented a unified

picture of the non-Euclidean geometry in which the findings of Lobachevsky and Riemann had appeared as particular cases.

3. John Herschel (1829, p. 222) called German all those who were united by language and behaviour. It is difficult, however, to unite thus Gauss and Bessel, or Karl Pearson and Fisher, or Markov and Liapunov.

Bibliography

Bachmann P. (1922), Über Gauss' zahlentheoretische Arbeiten. In Gauss, *Werke*, Bd. 10/2, No. 1.

Biermann K. R. (1983), C. F. Gauss als Mathematik- und Astronomiehistoriker. *Hist. Math.*, vol. 10, pp. 422 – 434. **S, G, 72.**

Eichhorn J. C., Editor (1805 – 1813), *Geschichte der Literatur von ihren Anfang bis auf die neuesten Zeiten*, Bde 1 – 6. Göttingen.

Gauss C. F. (1809, Latin), *Theorie der Bewegung* etc., Book 2, Section 3. In Gauss (1887, pp. 92 – 117).

... --- (1810), Untitled sections 102 and 103 in Bd. 3, No. 1, pp. 578 – 584 of Eichhorn (1805 – 1813).

--- (1823, Latin), English translation: Stewart (1995).

--- (1828), Bestimmung des Breitenunterschiede zwischen den Sternwarten von Göttingen und Altona etc. In Gauss (1887, pp. 152 – 189) and Gauss, W-9, 1903, pp. 5 – 64. **S, G, 72.**

--- (1887), *Abhandlungen zur Methode der kleinsten Quadrate*. Hrsg. A. Börsch, P. Simon. Vaduz, 1998.

--- (1975), *Briefwechsel mit H. C. Schumacher. Werke*, Ergänzungsreihe, Bd. 5. Hildesheim.

Herschel J. (1829), President, Address. *Mem. Astron. Soc. London*, vol. 4, pp. 217 – 224.

Liapunov A. M. (1895), P. L. Chebyshev. In Chebyshev P. L. (1946), *Izbrannye Matematicheskie Trudy* (Sel. Math. Works). Moscow – Leningrad, pp. 9 – 21.

May K. O. (1972), Gauss. *Dict. Scient. Biogr.*, vol. 5, vol. 298 – 315.

Novikov S. P. (2002, in Russian), The second half of the 20th century and its result: the crisis of the physical and mathematical association. *Istoriko-Matematicheskie Issledovania*, vol. 7 (42), pp. 326 – 356.

Sheynin O. (2012), New exposition of Gauss' final justification of least squares. *Math. Scientist*, vol. 37, pp. 147 – 148. *Silesian Stat. Rev.*, No. 12 (18), 2014, pp. 39 – 47.

Stewart G. W. (1995), *Theory of the Combination of Observations Least Subject to Error. C. F. Gauss*. Translation & Afterword. Philadelphia.

Wolf R. (1860), Joh. Heinrich Lambert von Mühlhausen. In author's *Biographien zur Kulturgeschichte des Schweiz*, 3. Cylus. Zürich, pp. 317 – 356.